

Modeling and Optimization of Adsorption of Heavy Metal Ions onto Local Activated Carbon

Abbas Sabah Thajeel

Chemical Engineering Department, College of Engineering, University of Basrah

Basrah, Iraq

E-mail: abbas_amery2002@yahoo.com

Received: April 22, 2013 Accepted: June 1, 2013 Published: July 1, 2013

doi:10.5296/ast.v1i2.3890 URL: <http://dx.doi.org/10.5296/ast.v1i2.3890>

Abstract

In this study, a mathematical model was constructed according to Central Composite Design method (CCD), which simulated the experimental work for adsorption of (Cu^{2+} , Fe^{3+} , Pb^{2+} and Zn^{2+}) in batch adsorption processes, where this model was studied the different effects of operational conditions and their impact on the efficiency of adsorption by using activated carbon produced from rice husk as local raw material which is low cost and available in huge quantities, and find a final form simulates practical experiences. Finally a mathematical model has been used as a software program (minitab16).

Keyword: Activated carbon, Adsorption, Modeling, Design of experiments, Heavy metal ions, Rice husk, Central composite design

1. Introduction

Heavy metal ions such as copper, iron, nickel, lead, etc. in the environment are of major concern due to their toxicity to many life forms. Unlike organic pollutants, which are susceptible to biological degradation, metal ions do not degrade into any harmless end products (Mohammadi et al., 2010) and tend to accumulate causing several diseases and health disorders in humans, and other living organisms (Rosa et al., 2008). Several industrial activities are important sources of environmental pollution due to their high content of several heavy metal ions (Dada et al., 2012). Wide range of various treatment techniques available for the removal of heavy metal ions from aqueous solutions such as ion exchange, biodegradation, oxidation, solvent extraction, chemical precipitation, flotation, biosorption, electrolytic recovery, membrane separation and adsorption have been reported to be used for removal of heavy metal ions from industrial effluents (Al-tameemi et al., 2012; Deosarkar, 2012). However, adsorption has been universally accepted as one of the most effective pollutant removal process, with low cost, ease in handling, low consumption of reagents, as well as scope for recovery of value added components through desorption and regeneration of adsorbent (Dada et al., 2012). Adsorption is collection of adsorbate on the surface of adsorbent due to force of attraction (Deosarkar, 2012). The practical applications of adsorption can be at separation and purification of liquid and gas mixtures, bulk chemicals, drying gases and liquids before loading them into industrial systems, removal of impurities from liquid and gas media, recovery of chemicals from industrial and vent gases and water purification (Prabakaran & Arivoli, 2012). Activated carbon is the most widely used adsorbent due to its excellent adsorption capability for heavy metals. However, the use of these methods is often limited due to the high cost, which makes them unfavorable for the needs of developing countries. Many reports have been investigated the low-cost adsorbents for Adsorption of heavy metals from aqueous solutions (Souag et al., 2009) such as date pits (Belhachemi et al., 2009) bamboo (Kannan & Veemaraj, 2009) oil palm fibre (Hameed et al., 2011; Nwabanne & Igbokwe, 2012), coconut shell (Satya et al., 1997), apricot stones (Philip & Girgis, 1996), sugar beet bagasse (Jaguaribe et al., 2000), waste tires (Teng et al., 2000; Juan et al., 2005; Mui et al., 2010), coconut husk, seed shell (Gueu et al., 2006), dates stones (Alhamed & Bamufleh, 2008), sun flower (Surchi, 2011), asphaltic carbon (Ambursa et al., 2011), Henna Leaves (Shanthi & Selvarajan, 2012). The intrinsic properties of activated carbon are dependent on the raw material source. The source of raw material was based on the need for developing low cost adsorbent for pollution control as well as reducing the effect of environmental degradation posed by agricultural waste (Itodo H. & Itodo A., 2010).

In this study, a simulation of batch adsorption processes was investigated by mathematical model for adsorption of heavy metal ions such as (Fe^{3+} , Zn^{2+} , Cu^{2+} and Pb^{2+}) from the (oil-water) polluted which comes out from the oil industry in Basrah city onto activated carbon produced from rice husk (RHAC) as local raw material which is low cost and available in huge quantities causing a pollutant problem.

2. Experimental Section

2.1 Materials

Zinc chloride with purity (97%) and sodium hydroxide with purity (97.5%) were supplied from THOMAS BAKER (Chemicals) Company. Copper chloride anhydrous with purity (99%) was supplied from BDH(Chemicals) Company. Iron nitrate (ferric nitrate) with purity (99%) was supplied from MERCK Company. Hydrogen chloride with purity (37%) was supplied from Scharlab.S.L Company. Nitrogen gas with purity (90%) and carbon dioxide gases with purity (95 - 99 %) were supplied from Basrah Factory. Rice husks were collected from Al-mushgab City Al-najaf Alashraf, Iraq, which had been discarded as waste from rice cultivation.

2.2 Adsorbent

Activated carbon produced from rice husk (RHAC) by physical method was used as an adsorbent material in this study, the preparation method was described following:

Initially, the (RH) were well washed with distilled water and dried in electrical oven for 24 hours. The carbonization step was carried out in electrical furnace for 2hr at 500°C and heating rate of 30 °C/min in absence air using nitrogen (N₂) at flow rate is 200 L/min.

In the activation step, the product from carbonization step was activated by passing carbon dioxide (CO₂) instead of nitrogen for 2hr at 700°C.

2.3 Preparation of Standard Solutions

The stock solutions of 1000 mg/L (ppm) of Cu²⁺, Fe³⁺, Pb²⁺ and Zn²⁺ were prepared by dissolving 2.1368 g of CuCl₂, 7.3073 g of Fe(NO₃)₃, 1.3557 g of PbCl₂ and 2.1273 g of ZnCl₂ in 1000 ml volumetric flasks and fill up to the mark with distilled water.

The diluted concentrations were prepared from stock solutions for carrying out experiments.

A certain volume (10 ml) of oil has been added to all above solutions with efficient agitation for simulated waste oil water.

2.4 Analyze a Sample of Heavy Metal Ions by Using Atomic Absorption Spectrophotometer (AAS)

The concentration of metal ions was measured by using atomic absorption spectrophotometer (BUCK Scientific, Model 210 VGP). In atomic absorption spectroscopy, metal atoms were vaporized into a flame, and the metal vapor absorbed radiation from the specific hollow cathode lamp in proportion to the number of atoms present. Beer's Law was followed in the part-per-million range (remember that ppm means mg of metal/liter of solution).

2.5 Adsorption Studies

Batch experiments were carried out by a (125 ml) flask. A certain weight of adsorbent material and (100 ml) of the solution prepared previously were added to the flask, and installed in the water bath (Mettler Gmbh Type WMB 22) at different temperature, see

Figure 1. The pH values were controlling by adding 0.1 N NaOH or 0.1 N HCl. A mixture with a different speed was mixed for 15 minutes using Variable-Speed Benchtop, model 5850, Eberbach. Finally, the mixtures were filtered through filter paper, and measurement of concentrations by Atomic Absorption.

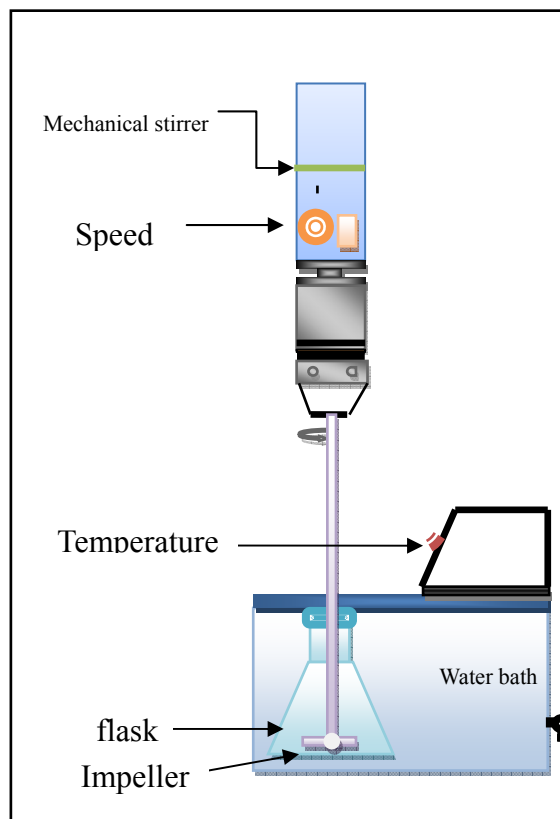


Figure 1. Schematic diagram of the batch adsorber (Al-Jomaa, 2011)

The removal percentage (R.P.%) which described the efficiency of adsorbent to adsorbed a heavy metal ions is calculated by following equation (BADMUS et al., 2007; Itodo et al., 2010):

$$R. P. \% = \frac{C_i - C_f}{C_i} * 100 \quad (1)$$

Where: C_i and C_f are the initial and final concentration in (ppm), respectively.

3. Results and Discussion

3.1 Modeling of Heavy Metal Ions Adsorption

Design of experiments (DOE) methods all involve: (1) carefully planning sets of input combinations to test using a random run order; then, (2) tests are performed and output values are recorded; (3) an interpolation method such as “regression” is then used to interpolate the outputs; and (4) the resulting prediction model is then used to predict new outputs for new

possible input combinations, DOE methods can be an important part of system optimization. These methods all involve the activities of experimental planning, conducting experiments, and fitting models to the outputs (Allen, 2006).

DOE methods are classified into several types, which included screening using fractional factorials, response surface methods (RSM), and robust design procedures. All of these DOE methods involve changing key input variable settings which are directly controllable (called factors) using carefully planned patterns, and then observing outputs (called responses) (Allen, 2006).

Response surface methodology is a collection of statistical and mathematical methods that are useful for the modeling and analyzing engineering problems. In this technique, the main objective is to optimize the response surface that is influenced by various process parameters. Response surface methodology also quantifies the relationship between the controllable input parameters and the obtained response surfaces (Tanet et al., 2007; Aslan & Cebeci, 2007).

The particular value of the variable is called the level of the factor. The combination of factors used in a particular experiment is called a treatment (Al-Badran, 2003; Ghadeer, 2009).

RSM methods are based on three types of design of experiments (DOE) matrices. First, central composite designs (CCD) are matrices corresponding to (at most) five level experimental plans from Box and Wilson (1951). Second, Box Behnken designs (BBD) are matrices corresponding to three level experimental plans from Box, Behnken (1960). Third, Allen et al. (2003) proposed methods based on so-called expected integrated mean squared error optimal (EIMSE-optimal) designs (Allen, 2006).

In this study, the central composite design was used to determine a models which described the relationship between the variables and the response.

The response which is the product (Y), is assumed to be a random variable (Lazic, 2004).

$$Y = f(X_1, X_2, X_3) + \text{Error} \quad (2)$$

A second degree polynomial equation was used if there is a curvature in the system, which given by Eq.(3) (Chen et al., 2011; Song et al., 2012; Daffalla et al., 2012):

$$Y_b = \beta_0 + \sum_{i=1}^k \beta_i X_i + \sum_{i=1}^k \beta_{ii} X_i^2 + \sum_{i < j}^k \sum_j^k \beta_{ij} X_i X_j \quad (3)$$

Where:

Y_b , X_i , β_0 , β_i , β_{ii} , β_{ij} and $X_{i,j}$ are the predicted response, independent variables, model constant, linear coefficients, the quadratic coefficients and cross product coefficients, the coded values of variables, respectively.

These second –order designs for k factors are composed of three sets of points (John, 1998):

(i) A 2^k factorial design with $X_i = \pm 1$, these are called the cube points. There are n_j of term.

(ii) A set of axial points. There are $n_\alpha = 2k$. Their coordinates are $(\pm\alpha, 0, 0, \dots)$, $(\dots, 0, \pm\alpha, 0, \dots)$ and $(\dots, 0, 0, \pm\alpha)$, where α is the distance of the axial point from center and makes the design rotatable (Tan et al., 2007).

(iii) n_0 center points, which used to determine the experimental error and the reproducibility of the data.

The number of experiments (N) needed was calculated by the following equation (Lazic, 2004; John, 1998):

$$N = n_j + n_\alpha + n_0 = 2^k + 2*k + n_0 \quad (4)$$

For five factors a second order polynomial mathematical model is described by the following equation:

$$Y_b = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{33} X_3^2 + \beta_{44} X_4^2 + \beta_{55} X_5^2 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5 \quad (5)$$

The relationship between the coded levels and the corresponding actual variables is represented by the equation:

$$X_{coded} = \frac{(x_{actual} - x_{center})}{(x_{center} - x_{minimum})} \quad (6)$$

The low and height values of each variable in batch system were listed in Table 1:

Table 1. The low and height values of variables in batch system

Variable	Simple	Low	Height
PH	X_1	5	9
Temperature \hat{C}	X_2	22	50
Doseg	X_3	2	4
Concentration ppm	X_4	21	50
rpm	X_5	350	750

The relationship between the coded variable (X) and the corresponding real variable(x) as following :

$$X_1 = \frac{(x_1 - 7)}{2}, X_2 = \frac{(x_2 - 36)}{14}, X_3 = \frac{(x_3 - 3)}{1}$$

$$X_4 = \frac{(x_4 - 35.5)}{14.5}, X_5 = \frac{(x_5 - 550)}{200} \quad (7)$$

The central composite design can be made to be rotatable by choosing $\alpha = 2^{K/4}$ when a complete factorial is used. For five factors:

$$\alpha = 2^{5/4} = 2.378 \quad (8)$$

Table 2. The coded & uncoded (actual) values of variables in batch system

No.	X1	X2	X3	X4	X5	X1 PH	X2Temp.	X3dose	X5CONC.	X5 rpm
1	1	1	1	1	1	9	50	4	50	750
2	-1	1	1	1	1	5	50	4	50	750
3	1	-1	1	1	1	9	22	4	50	750
4	-1	-1	1	1	1	5	22	4	50	750
5	1	1	-1	1	1	9	50	2	50	750
6	-1	1	-1	1	1	5	50	2	50	750
7	1	-1	-1	1	1	9	22	2	50	750
8	-1	-1	-1	1	1	5	22	2	50	750
9	1	1	1	-1	1	9	50	4	21	750
10	-1	1	1	-1	1	5	50	4	21	750
11	1	-1	1	-1	1	9	22	4	21	750
12	-1	-1	1	-1	1	5	22	4	21	750
13	1	1	-1	-1	1	9	50	2	21	750
14	-1	1	-1	-1	1	5	50	2	21	750
15	1	-1	-1	-1	1	9	22	2	21	750
16	-1	-1	-1	-1	1	5	22	2	21	750
17	1	1	1	1	-1	9	50	4	50	350
18	-1	1	1	1	-1	5	50	4	50	350
19	1	-1	1	1	-1	9	22	4	50	350
20	-1	-1	1	1	-1	5	22	4	50	350
21	1	1	-1	1	-1	9	50	2	50	350
22	-1	1	-1	1	-1	5	50	2	50	350
23	1	-1	-1	1	-1	9	22	2	50	350
24	-1	-1	-1	1	-1	5	22	2	50	350
25	1	1	1	-1	-1	9	50	4	21	350
26	-1	1	1	-1	-1	5	50	4	21	350
27	1	-1	1	-1	-1	9	22	4	21	350
28	-1	-1	1	-1	-1	5	22	4	21	350
29	1	1	-1	-1	-1	9	50	2	21	350
30	-1	1	-1	-1	-1	5	50	2	21	350
31	1	-1	-1	-1	-1	9	22	2	21	350
32	-1	-1	-1	-1	-1	5	22	2	21	350
33	$-\alpha$	0	0	0	0	2.243	36	3	35.5	550
34	α	0	0	0	0	11.76	36	3	35.5	550
35	0	$-\alpha$	0	0	0	7	2.702	3	35.5	550
36	0	α	0	0	0	7	69.3	3	35.5	550
37	0	0	$-\alpha$	0	0	7	36	0.6216	35.5	550
38	0	0	α	0	0	7	36	5.3784	35.5	550
39	0	0	0	$-\alpha$	0	7	36	3	1.013	550
40	0	0	0	α	0	7	36	3	69.99	550
41	0	0	0	0	$-\alpha$	7	36	3	35.5	74.32
42	0	0	0	0	α	7	36	3	35.5	1026
43	0	0	0	0	0	7	36	3	35.5	550
44	0	0	0	0	0	7	36	3	35.5	550
45	0	0	0	0	0	7	36	3	35.5	550
46	0	0	0	0	0	7	36	3	35.5	550
47	0	0	0	0	0	7	36	3	35.5	550
48	0	0	0	0	0	7	36	3	35.5	550
49	0	0	0	0	0	7	36	3	35.5	550
50	0	0	0	0	0	7	36	3	35.5	550
51	0	0	0	0	0	7	36	3	35.5	550
52	0	0	0	0	0	7	36	3	35.5	550

The regression coefficients are determined by equations (Lazic, 2004):

$$\beta_{o=a_1} \sum_1^N Y_b - a_2 \sum_1^K \sum_1^N X_i^2 Y_b$$

$$\beta_{i=a_3} \sum_1^N X_i Y_b$$

$$\beta_{ij=a_4} \sum_1^N X_i X_j Y_b$$

$$\beta_{ii=a_5} \sum_1^N X_i^2 Y_b + a_6 \sum_1^K \sum_1^N X_i^2 Y_b - a_7 \sum_1^N Y_b \quad (9)$$

Where: $a_1 \dots \dots a_7$ are coefficients as determined from Table 3.

Table 3. Coefficients values $a_1 \dots \dots a_7$

0.0988	0.0191	0.0231	0.0312	0.0156	0.0015	0.0191
--------	--------	--------	--------	--------	--------	--------

The results of experimental work of adsorption Cu^{2+} , Fe^{3+} , Pb^{2+} and Zn^{2+} ions onto RHAC in batch processes are listed in Table A.1 Appendix A.

For Cu^{2+} ions were adsorbed on the RHAC in batch process:

Table 4. The calculated regression coefficients for Cu^{2+} ions

No.	Y_b	$X_1 Y_b$	$X_2 Y_b$	$X_3 Y_b$	$X_4 Y_b$	$X_5 Y_b$	$X_1 X_2 Y_b$	$X_1 X_3 Y_b$
1	81.23938	81.23938	81.23938	81.23938	81.23938	81.23938	81.23938	81.23938
2	84.54624	-84.5462	84.54624	84.54624	84.54624	84.54624	-84.5462	-84.5462
3	74.00122	74.00122	-74.0012	74.00122	74.00122	74.00122	-74.0012	74.00122
4	76.60166	-76.6017	-76.6017	76.60166	76.60166	76.60166	76.60166	-76.6017
5	69.63233	69.63233	69.63233	-69.6323	69.63233	69.63233	69.63233	-69.6323
6	73.71796	-73.718	73.71796	-73.718	73.71796	73.71796	-73.718	73.71796
7	64.84563	64.84563	-64.8456	-64.8456	64.84563	64.84563	-64.8456	-64.8456
8	67.21829	-67.2183	-67.2183	-67.2183	67.21829	67.21829	67.21829	67.21829
9	90.37154	90.37154	90.37154	90.37154	-90.3715	90.37154	90.37154	90.37154
10	92.9487	-92.9487	92.9487	92.9487	-92.9487	92.9487	-92.9487	-92.9487
11	85.91279	85.91279	-85.9128	85.91279	-85.9128	85.91279	-85.9128	85.91279
12	87.62928	-87.6293	-87.6293	87.62928	-87.6293	87.62928	87.62928	-87.6293
13	80.79923	80.79923	80.79923	-80.7992	-80.7992	80.79923	80.79923	-80.7992
14	82.03279	-82.0328	82.03279	-82.0328	-82.0328	82.03279	-82.0328	82.03279

15	78.91772	78.91772	-78.9177	-78.9177	-78.9177	78.91772	-78.9177	-78.9177
16	79.54887	-79.5489	-79.5489	-79.5489	-79.5489	79.54887	79.54887	79.54887
17	83.60205	83.60205	83.60205	83.60205	83.60205	-83.602	83.60205	83.60205
18	85.7452	-85.7452	85.7452	85.7452	85.7452	-85.7452	-85.7452	-85.7452
19	73.71796	73.71796	-73.718	73.71796	73.71796	-73.718	-73.718	73.71796
20	76.22091	-76.2209	-76.2209	76.22091	76.22091	-76.2209	76.22091	-76.2209
21	70.99306	70.99306	70.99306	-70.9931	70.99306	-70.9931	70.99306	-70.9931
22	74.11563	-74.1156	74.11563	-74.1156	74.11563	-74.1156	-74.1156	74.11563
23	65.5934	65.5934	-65.5934	-65.5934	65.5934	-65.5934	-65.5934	-65.5934
24	68.79341	-68.7934	-68.7934	-68.7934	68.79341	-68.7934	68.79341	68.79341
25	91.42334	91.42334	91.42334	91.42334	-91.4233	-91.4233	91.42334	91.42334
26	92.69899	-92.699	92.69899	92.69899	-92.699	-92.699	-92.699	-92.699
27	84.44523	84.44523	-84.4452	84.44523	-84.4452	-84.4452	-84.4452	84.44523
28	87.91037	-87.9104	-87.9104	87.91037	-87.9104	-87.9104	87.91037	-87.9104
29	81.41817	81.41817	81.41817	-81.4182	-81.4182	-81.4182	81.41817	-81.4182
30	81.72603	-81.726	81.72603	-81.726	-81.726	-81.726	-81.726	81.72603
31	77.64412	77.64412	-77.6441	-77.6441	-77.6441	-77.6441	-77.6441	-77.6441
32	79.86298	-79.863	-79.863	-79.863	-79.863	-79.863	79.86298	79.86298
33	65.16427	-154.987	0	0	0	0	0	0
34	47.34202	112.5987	0	0	0	0	0	0
35	84.75804	0	-201.589	0	0	0	0	0
36	93.10076	0	221.4318	0	0	0	0	0
37	51.82756	0	0	-123.267	0	0	0	0
38	97.12532	0	0	231.0038	0	0	0	0
39	100	0	0	0	-237.841	0	0	0
40	87.49923	0	0	0	208.109	0	0	0
41	78.67269	0	0	0	0	-187.116	0	0
42	93.75402	0	0	0	0	222.9855	0	0
43	96.60561	0	0	0	0	0	0	0
44	96.06727	0	0	0	0	0	0	0
45	97.94627	0	0	0	0	0	0	0
46	96.95424	0	0	0	0	0	0	0
47	97.46075	0	0	0	0	0	0	0
48	96.2487	0	0	0	0	0	0	0
49	97.21003	0	0	0	0	0	0	0
50	96.95424	0	0	0	0	0	0	0
51	95.32283	0	0	0	0	0	0	0
52	95.03638	0	0	0	0	0	0	0
Sum	4310.925	-79.1487	107.9892	259.8919	-194.438	29.92239	0.655258	-2.41555

Table 4.Continued

$X_1X_4Y_b$	$X_1X_5Y_b$	$X_2X_3Y_b$	$X_2X_4Y_b$	$X_2X_5Y_b$	$X_3X_4Y_b$	$X_3X_5Y_b$	$X_4X_5Y_b$	$X_1^2Y_b$
81.23938	81.23938	81.23938	81.23938	81.23938	81.23938	81.23938	81.23938	81.23938

-84.5462	-84.5462	84.54624	84.54624	84.54624	84.54624	84.54624	84.54624	84.54624
74.00122	74.00122	-74.0012	-74.0012	-74.0012	74.00122	74.00122	74.00122	74.00122
-76.6017	-76.6017	-76.6017	-76.6017	-76.6017	76.60166	76.60166	76.60166	76.60166
69.63233	69.63233	-69.6323	69.63233	69.63233	-69.6323	-69.6323	69.63233	69.63233
-73.718	-73.718	-73.718	73.71796	73.71796	-73.718	-73.718	73.71796	73.71796
64.84563	64.84563	64.84563	-64.8456	-64.8456	-64.8456	-64.8456	64.84563	64.84563
-67.2183	-67.2183	67.21829	-67.2183	-67.2183	-67.2183	-67.2183	67.21829	67.21829
-90.3715	90.37154	90.37154	-90.3715	90.37154	-90.3715	90.37154	-90.3715	90.37154
92.9487	-92.9487	92.9487	-92.9487	92.9487	-92.9487	92.9487	-92.9487	92.9487
-85.9128	85.91279	-85.9128	85.91279	-85.9128	-85.9128	85.91279	-85.9128	85.91279
87.62928	-87.6293	-87.6293	87.62928	-87.6293	-87.6293	87.62928	-87.6293	87.62928
-80.7992	80.79923	-80.7992	-80.7992	80.79923	80.79923	-80.7992	-80.7992	80.79923
82.03279	-82.0328	-82.0328	-82.0328	82.03279	82.03279	-82.0328	-82.0328	82.03279
-78.9177	78.91772	78.91772	78.91772	-78.9177	78.91772	-78.9177	-78.9177	78.91772
79.54887	-79.5489	79.54887	79.54887	-79.5489	79.54887	-79.5489	-79.5489	79.54887
83.60205	-83.602	83.60205	83.60205	-83.602	83.60205	-83.602	-83.602	83.60205
-85.7452	85.7452	85.7452	85.7452	-85.7452	85.7452	-85.7452	-85.7452	85.7452
73.71796	-73.718	-73.718	-73.718	73.71796	73.71796	-73.718	-73.718	73.71796
-76.2209	76.22091	-76.2209	-76.2209	76.22091	76.22091	-76.2209	-76.2209	76.22091
70.99306	-70.9931	-70.9931	70.99306	-70.9931	-70.9931	70.99306	-70.9931	70.99306
-74.1156	74.11563	-74.1156	74.11563	-74.1156	-74.1156	74.11563	-74.1156	74.11563
65.5934	-65.5934	65.5934	-65.5934	65.5934	-65.5934	65.5934	-65.5934	65.5934
-68.7934	68.79341	68.79341	-68.7934	68.79341	-68.7934	68.79341	-68.7934	68.79341
-91.4233	-91.4233	91.42334	-91.4233	-91.4233	-91.4233	-91.4233	91.42334	91.42334
92.69899	92.69899	92.69899	-92.699	-92.699	-92.699	-92.699	92.69899	92.69899
-84.4452	-84.4452	-84.4452	84.44523	84.44523	-84.4452	-84.4452	84.44523	84.44523
87.91037	87.91037	-87.9104	87.91037	87.91037	-87.9104	-87.9104	87.91037	87.91037
-81.4182	-81.4182	-81.4182	-81.4182	-81.4182	81.41817	81.41817	81.41817	81.41817
81.72603	81.72603	-81.726	-81.726	-81.726	81.72603	81.72603	81.72603	81.72603
-77.6441	-77.6441	77.64412	77.64412	77.64412	77.64412	77.64412	77.64412	77.64412
79.86298	79.86298	79.86298	79.86298	79.86298	79.86298	79.86298	79.86298	79.86298
0	0	0	0	0	0	0	0	368.6235
0	0	0	0	0	0	0	0	267.806
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
-9.90841	-0.28775	24.12526	25.05191	-6.92138	9.374568	0.920755	-8.01056	3182.304

Table 4. Continued

$X_2^2 Y_b$	$X_3^2 Y_b$	$X_4^2 Y_b$	$X_5^2 Y_b$	Y_m	$(Y_b - Y_m)^2$
81.23938	81.23938	81.23938	81.23938	87.11434	34.51512
84.54624	84.54624	84.54624	84.54624	91.51709	48.59274
74.00122	74.00122	74.00122	74.00122	79.44759	29.66286
76.60166	76.60166	76.60166	76.60166	83.93212	53.7356
69.63233	69.63233	69.63233	69.63233	73.11022	12.09568
73.71796	73.71796	73.71796	73.71796	77.21151	12.20492
64.84563	64.84563	64.84563	64.84563	68.4543	13.02248
67.21829	67.21829	67.21829	67.21829	72.63737	29.36644
90.37154	90.37154	90.37154	90.37154	95.0673	22.05009
92.9487	92.9487	92.9487	92.9487	98.23348	27.92887
85.91279	85.91279	85.91279	85.91279	90.52702	21.29116
87.62928	87.62928	87.62928	87.62928	93.77498	37.76972
80.79923	80.79923	80.79923	80.79923	82.23312	2.056057
82.03279	82.03279	82.03279	82.03279	85.09785	9.394581
78.91772	78.91772	78.91772	78.91772	80.70368	3.189654
79.54887	79.54887	79.54887	79.54887	83.65018	16.82076
83.60205	83.60205	83.60205	83.60205	86.62418	9.133279
85.7452	85.7452	85.7452	85.7452	90.99102	27.51868
73.71796	73.71796	73.71796	73.71796	78.09364	19.14658
76.22091	76.22091	76.22091	76.22091	82.54226	39.95944
70.99306	70.99306	70.99306	70.99306	72.73497	3.034249
74.11563	74.11563	74.11563	74.11563	76.80035	7.20774
65.5934	65.5934	65.5934	65.5934	67.21526	2.630444
68.79341	68.79341	68.79341	68.79341	71.36242	6.599787
91.42334	91.42334	91.42334	91.42334	93.57742	4.640028
92.69899	92.69899	92.69899	92.69899	96.70769	16.06966
84.44523	84.44523	84.44523	84.44523	88.17336	13.89892
87.91037	87.91037	87.91037	87.91037	91.38541	12.07587
81.41817	81.41817	81.41817	81.41817	80.85815	0.313622
81.72603	81.72603	81.72603	81.72603	83.68697	3.845272
77.64412	77.64412	77.64412	77.64412	78.46492	0.673716

79.86298	79.86298	79.86298	79.86298	81.37551	2.287774
0	0	0	0	62.20889	8.734268
0	0	0	0	53.51183	38.0665
479.4622	0	0	0	84.55103	0.042851
526.6556	0	0	0	96.41716	10.99853
0	293.1799	0	0	61.77557	98.963
0	549.4218	0	0	90.33316	46.13347
0	0	565.6834	0	105.9792	35.75113
0	0	494.9686	0	84.61391	8.325074
0	0	0	445.0384	86.12845	55.5883
0	0	0	530.351	89.41639	18.81502
0	0	0	0	96.43361	0.029582
0	0	0	0	96.43361	0.134211
0	0	0	0	96.43361	2.288123
0	0	0	0	96.43361	0.27105
0	0	0	0	96.43361	1.055012
0	0	0	0	96.43361	0.034194
0	0	0	0	96.43361	0.602821
0	0	0	0	96.43361	0.27105
0	0	0	0	96.43361	1.233846
0	0	0	0	96.43361	1.952264
3551.992	3388.476	3606.526	3521.264	4442.577	872.0221

Table 4.Continued

β_{12}	β_{13}	β_{14}	β_{15}	β_{23}	β_{24}	β_{25}	β_{34}	β_{35}	β_{45}	β
0.02044	-0.075	-0.30	-0.0089	0.752	0.781	-0.215	0.292	0.028	-0.24	96.433
β_1	β_2	β_3	β_4	β_5	β_{11}	β_{22}	β_{33}	β_{44}	β_{55}	
-1.828	2.494	6.003	-4.491	0.691	-6.818	-1.051	-3.602	-0.201	-1.5311	

The adequacy of regression model have been checked with Fisher's (F_R) value and tabular value (F_T) by following equations:

$$F_R = \frac{S_{AD}^2}{S_Y^2} \quad (10)$$

For calculation of S_{AD}^2 , the expression:

$$S_{AD}^2 = \frac{SS_R - SS_E}{f_{AD}} = \frac{\sum_1^N (Y_b - \hat{Y}_m)^2 - (Y_{oj} - \bar{Y}_o)^2}{N - (n_o - 1) - \lambda} \quad (11)$$

$$S_Y^2 = \frac{S_Y^2}{N} \quad (12)$$

$$S_Y^2 = \frac{\sum_1^{n_o} (Y_{oj} - \bar{Y}_o)^2}{n_o - 1} \quad (13)$$

Knowledge of S_{AD}^2 and $S_{\bar{Y}}^2$ facilitates determination of both calculating the value of Fisher's criterion and simultaneously of the tabular value by which we may compare and accept or reject the hypothesis of lack of fit of the regression model.

Where:

SS_E is sum of squares of reproducibility variance.

SS_R is residual sum of squares.

Y_b is outcome of each trial.

Y_m is calculated response value from regression equation.

Y_{oj} is the outcome of one trial in null point.

\bar{Y}_o is average of replications in null point.

The rotatability conditions is defined by following relations:

$$f_{AD} = N - \lambda - (n_o - 1) \quad (14)$$

For second order regression models:

$$\lambda = \frac{(K+2)(k+1)}{2} \quad (15)$$

The value degree of freedom (f_E) is calculated by following equation:

$$f_E = N(n - 1) \quad (16)$$

Table 5. (F_R) and (F_T) values

Y_{oj}	$(Y_{oj} - \bar{Y}_o)^2$
96.60561	0.0006239
96.06727	0.263544
97.94627	1.8649653
96.95424	0.1395831
97.46075	0.774613
96.2487	0.1101788
97.21003	0.3961424
96.95424	0.1395831
95.32283	1.5820683
95.03638	2.3847118
sum	sum
965.8063	7.6560138

\bar{Y}_o	S_Y^2	S_{AD}^2	S_Y^2	F_R	f_{AD}	f_E	F_T
96.58063	0.8506682	39.28937	44.23475	0.888201	22	468	2.31

A tabular value F_T is obtained for $f_{AD} = 22$ and $f_E = 52(10-1) = 468$ and $1-\alpha = 99\%$, from Table A.2 in Appendix A (Lazic, 2004).

The regression model is adequate with 99% confidence because $F_R < F_T$ (Fisher's value < tabular value).

A check of significance of regression coefficients is brought down to determining their confidence intervals and their comparison to absolute values of regression coefficients. The rule is (Lazic, 2004):

A regression coefficient is statistically significant if its absolute value is higher than the confidence interval.

When estimating the significance of regression coefficients, these equations are used:

$$S_{\beta^o}^2 = \frac{2A\lambda(K+2)}{N} S_Y^2 \quad (17)$$

$$S_{\beta_i}^2 = \frac{S_Y^2}{N-n^o} \quad (18)$$

$$S_{\beta_{ij}}^2 = \frac{C^2}{N} S_Y^2 \quad (19)$$

$$S_{\beta_{ii}}^2 = \frac{AC^2[(K+2)\lambda - (K-2)]}{N} S_Y^2 \quad (20)$$

$$A = \frac{1}{2\lambda[(K+2)\lambda - K]} \quad (21)$$

$$C = \frac{N}{N-n^o} \quad (22)$$

Where:

S_{β^o} , S_{β_i} , $S_{\beta_{ij}}$ and $S_{\beta_{ii}}$ are variance of regression coefficients which associated error mean squares in determining regression coefficients β^o , β_i , β_{ij} and $S_{\beta_{ii}}$.

In the case of second-order designs of regression coefficient significances, they are checked by using:

Table 6. $\Delta\beta_o, \Delta\beta_i, \Delta\beta_{ij}$ and $\Delta\beta_{ii}$ values

S_{β_o}	0.016359	$\Delta\beta_o = \mp 2S_{\beta_o}$	∓ 0.255805
S_{β_i}	0.0236297	$\Delta\beta_i = \mp 2S_{\beta_i}$	∓ 0.307439
$S_{\beta_{ij}}$	0.0265834	$\Delta\beta_{ij} = \mp 2S_{\beta_{ij}}$	∓ 0.326088
$S_{\beta_{ii}}$	0.0862951	$\Delta\beta_{ii} = \mp 2S_{\beta_{ii}}$	∓ 0.587521

A check of statistical significance of regression coefficients indicates that regression coefficients $\beta_o, \beta_i, \beta_{23}, \beta_{24}, \beta_{11}, \beta_{22}, \beta_{33}$ and β_{55} are statistically significant, while the other coefficients are insignificant. The final form of the second order regression model with 99% confidence may be given in the form:

$$Y_b = 96.433 - 1.828X_1 + 2.494X_2 + 6.003X_3 - 4.491X_4 + 0.691X_5 - 6.818X_1^2 - 1.051X_2^2 - 3.602X_3^2 - 1.5311X_5^2 + 0.752X_2X_3 + 0.781X_2X_4 \quad (23)$$

The same above calculations method are used to determine the models of Fe^{3+}, Pb^{2+} and Zn^{2+} in batch process. The regression coefficients and final model equations are show below.

For Fe^{3+} ions were adsorbed on the RHAC in batch process:

 Table 7. The calculated regression coefficients for Fe^{3+} ions

β_1										
1.544	3.275	6.2902	-4.071	0.458	-5.782	-0.689	-2.315	0.1423	-1.383	
									β_{45}	β
0.252	-0.50	0.021	0.175	-0.071	0.348	-0.155	2.007	0.073	0.064	93.962
<hr/>										
\bar{Y}_o	S_V^2	S_{AD}^2	S_V^2	F_R	f_{AD}	f_E	F_T			
94.06706	1.9549847	31.14317	101.6592	0.306349	22	468	2.31			

A check of statistical significance of regression coefficients indicates that regression coefficients $\beta_o, \beta_i, \beta_{13}, \beta_{34}, \beta_{11}, \beta_{33}$ and β_{55} are statistically significant, while the other coefficients are insignificant. The final form of the second order regression model with 99% confidence may be given in the form:

$$Y_b = 93.962 + 1.544X_1 + 3.275X_2 + 6.2902X_3 - 4.071X_4 + 0.458X_5 - 5.782X_1^2 - 2.315X_3^2 - 1.383X_5^2 + 0.752X_1X_3 + 0.781X_3X_4 \quad (24)$$

For Pb^{2+} ions were adsorbed on the RHAC in batch process:

Table 8. The calculated regression coefficients for Pb^{2+} ions

β_1									
-1.564	1.609	5.722	-4.82	0.827	-1.175	0.361	-3.078	0.9606	-0.6859
									β_{45}
0.1344	-0.26	0.454	-0.074	0.187	0.148	0.05385	1.579	0.115	88.92243
\bar{Y}_o	S_Y^2	S_{AD}^2	S_Y^2	F_R	f_{AD}	f_E	F_T		
88.79165	4.0477878	44.59781	210.485	0.211881	22	468	2.31		

A check of statistical significance of regression coefficients indicates that regression coefficients β_{\square} , β_i , β_{11} , β_{33} and β_{34} are statistically significant, while the other coefficients are insignificant. The final form of the second order regression model with 99% confidence may be given in the form:

$$Y_b = 88.922 - 1.564X_1 + 1.609X_2 + 5.722X_3 - 4.82X_4 + 0.827X_5 - 1.175X_1^2 - 3.078X_3^2 + 1.579X_3X_4 \quad (25)$$

For Zn^{2+} ions were adsorbed on the RHAC in batch process:

Table 9. The calculated regression coefficients for Zn^{2+} ions

β_1									
5.467	2.961	7.053	-8.78	0.424	-5.396	-0.35678	-2.539	1.1253	-1.882
									β_{45}
0.3605	-0.28	1.583	-0.336	0.166	0.817	0.0218	-1.14	0.194	73.368
\bar{Y}_o	S_Y^2	S_{AD}^2	S_Y^2	F_R	f_{AD}	f_E	F_T		
73.89135	2.9076597	57.52014	151.1983	0.380428	22	468	2.31		

A check of statistical significance of regression coefficients indicates that regression coefficients β_{\square} , β_i , β_{14} , β_{24} , β_{34} , β_{11} , β_{33} , β_{44} and β_{55} are statistically significant, while the other coefficients are insignificant. The final form of the second order regression model with 99% confidence may be given in the form:

$$Y_b = 73.368 + 5.467X_1 + 2.961X_2 + 7.053X_3 - 8.78X_4 + 0.424X_5 - 5.396X_1^2 - 2.539X_3^2 + 1.1253X_4^2 - 1.882X_5^2 + 1.583X_1X_4 + 0.817X_2X_4 - 1.14X_3X_5 \quad (26)$$

3.2 Modeling Using Minitab Software

Minitab is a statistical software, it was developed by Minitab Inc. (USA). Minitab 16.1.0, was used in this study to determine the models of adsorption of heavy metal ions on RHAC.

The calculations and results of models which were determined by this program are listed in Appendix (B), clarification and explanation of the tables and calculations are shown below.

The coefficients table is listed the estimated coefficients for the variables.

Regression examines the relationship between a response and variables. In order to determine whether or not the observed relationship between the response and variables is statistically significant, need to:

Identify the coefficient p-values: the coefficient value for P (p-value) tells whether or not the association between the response and variables is statistically significant. Compare the coefficient p-values to α -level: if the p-value is smaller than the α -level, the association is statistically significantly.

P regression was used to test the hypothesis that all the coefficients in the model are zero. A smaller p-value than a pre-selected selected α -level implies that at least one coefficient in the model is not zero.

P lack of fit was used to test whether the model fits the data well. A smaller p-value than α -level indicates that might need to consider higher order terms of existing predictors, or additional predictors, to get a better fit of the data.

A list of the standard errors for the estimated constant and the estimated coefficient. A standard error for an estimated coefficient measures the precision of the estimate. The smaller the standard error, the more precise the estimate.

S is measured in the units of the response variable and represents the standard distance data values fall from the regression line. For a given study, the better equation that predicts the response, the lower S is.

Minitab displays the coefficients in uncoded units in addition to coded units. For each term in the model, there is a coefficient. Use these coefficients to construct an equation representing the relationship between the response and the factors.

To use this equation, put in the uncoded (actual) factor values and calculate the variables response. Because these coefficients are estimated using uncoded units, putting coded factor values into this equation would produce incorrect predictions about yield.

Note: the above clarification and explanation were quoted from the help of program.

The final model equations were calculated by this program are shown below:

For Cu^{2+} ions were adsorbed on the RHAC in batch process:

$$Y_b = 96.3394 - 1.8273X_1 + 2.4932X_2 + 6.0X_3 - 4.489X_4 - 7.41X_1^2 - 1.6345X_2^2 - 4.189X_3^2 - 2.1146X_5^2 \quad (27)$$

For Fe^{3+} ions were adsorbed on the RHAC in batch process:

$$Y_b = 93.868 + 1.543X_1 + 3.27X_2 + 6.28X_3 - 4.069X_4 - 6.38X_1^2 - 1.278X_2^2 - 2.906X_3^2 - 1.973X_5^2 + 2.01X_3X_4 \quad (28)$$

For Pb^{2+} ions were adsorbed on the RHAC in batch process :

$$Y_b = 88.827 - 1.56X_1 + 1.608X_2 + 5.7198X_3 - 4.82X_4 - 1.778X_1^2 - 3.684X_2^2 - 1.288X_5^2 \quad (29)$$

For Zn^{2+} ions were adsorbed on the RHAC in batch process:

$$Y_b = 73.295 + 5.464X_1 + 2.96X_2 + 7.049X_3 - 8.781X_4 - 5.855X_1^2 - 2.993X_3^2 - 2.3357X_5^2 \quad (30)$$

3.3 Validity of Models

The validity of each equations of model can be tested by the Sum of Squared Errors (SSE %), the sum of squared errors was determined by following (Tanet al., 2007):

$$SSE \% = \sqrt{\frac{\sum(RP_{exp} - RP_{cal})^2}{N}} \quad (31)$$

The lower value of SSE is indicate the better, which indicates that the best model can be chosen.

Table 10. The values of sum of squared errors

H.M.I.	Cu^{2+}		Fe^{3+}		Pb^{2+}		Zn^{2+}	
Models	CCD	Minitab	CCD	Minitab	CCD	Minitab	CCD	Minitab
SSE %	4.252	3.370	3.676	2.523	4.304	6.715	4.983	4.973

Where:

N is the number of data points. RP_{exp} is the values of removal percentage from experimental work. RP_{cal} is the values of removal percentage which calculated from models. H.M.I. is heavy metal ions.

4. Conclusion

A mathematical model was constructed according to Central Composite Design method (CCD) and a software program (minitab16). These models were simulated experimental work for adsorption of (Cu^{2+} , Fe^{3+} , Pb^{2+} and Zn^{2+}) in batch adsorption processes using activated carbon produced from rice husk as local raw material which is low cost and available in huge quantities causing a pollutant problem. Final modeling equations were well simulated experimental work with very little deviation by Fisher's testing (1%), as well as the results of equations derived using (minitab16).

References

- Al-Badran, F. (2003). *Chlorinated polyethylene Wax from Industrial Waste: Production, Characterization, Modeling and Kinetic Study*. (M.Sc. Thesis). Chemical Engineering Department, College of Engineering, Basrah University.
- Alhamed, Y. A., & Bamufleh, H. S. (2008). Sulfur Removal From Model Diesel Fuel Using Granular Activated Carbon From Dates' Stones Activated by $ZnCl_2$. *Journal of Fuel*, 88, 87-94. <http://dx.doi.org/10.1016/j.fuel.2008.07.019>
- Al-Jomaa, S. F. (2011). *A new local adsorbent for the treatment of an industrial wastewater from toxic metals*. (M. Sc. Thesis). Chemical Engineering Department, College of Engineering, Basrah University.
- Allen, T. T. (2006). *Introduction to Engineering Statistics and Six Sigma*. Springer-Verlag London Limited.
- Al-tameemi, I. A., Nasser, T., & Thuraya, M. A. (2012). Removal of pb (II), cd (II) Ions onto dried Conocarpus erectus leaves using batch adsorption and study the adsorption thermodynamics. *Journal of Chem. Pharm. Res.*, 12, 4961-4968.
- Ambursa, M. M., Faruk, U. Z., Uba, A., Sahabi, D. M., Atiku, F. A., & Koko, R. A. (2011). Comparative efficiency of base activated carbon and acid activated carbon for sorption of heavy metals from waste water. *Journal of Chem. Pharm. Res.*, 3(6), 732-741.
- Aslan, N., & Cebeci, Y. (2007). Application of Box-Behnken design and response surface methodology for modeling of some Turkish coals. *Journal of Fuel*, 86, 90-97. <http://dx.doi.org/10.1016/j.fuel.2006.06.010>
- Badmus, M. A. O., Audu, T. O. K., & Anyata, B. U. (2007). Removal of Lead Ion from Industrial Wastewaters by Activated Carbon Prepared from Periwinkle Shells. *Turkish, Journal Eng. Env. Sci.*, 31, 251-263.
- Belhachemi, M., Rios, R., Addoun, F., Albero, J. S., Escibano, A. S., & Reinoso, F. R. (2009). Preparation of activated carbon from date pits: Effect of the activation agent and liquid phase oxidation. *Journal of Analytical and Applied Pyrolysis*, 86, 168-172. <http://dx.doi.org/10.1016/j.jaap.2009.05.004>
- Chen, Y., Zhu, Y., Wang, Z., Li, Y., Wang, L., Lili, Ding, X. G., Ma, Y., & Guo, Y. (2011).

Application studies of activated carbon derived from rice husks produced by chemical-thermal process. *Advances in Colloid and Interface Science*, 163, 39-52. <http://dx.doi.org/10.1016/j.cis.2011.01.006>

Dada, A. O., Olalekan, A. P., Olatunya, A. M., & Dada, O. (2012). Langmuir, Freundlich, Temkin and Dubinin–Radushkevich Isotherms Studies of Equilibrium Sorption of Zn^{2+} Unto Phosphoric Acid Modified Rice Husk. *IOSR Journal of Applied Chemistry (IOSR-JAC)*, 3(1), 38-45.

Daffalla, S. B., Mukhtar, H., & Shaharun, M. S. (2012). Effect of Organic and Inorganic Acid Pretreatment on Structural Properties of Rice Husk and Adsorption Mechanism of Phenol. *International Journal of Chemical and Environmental Engineering*, 3(3), 192-200.

Deosarkar, S. D. (2012). Cu(II) Adsorption from Aqueous Solution by Punicagranatum L. Husk. *Journal of Chem. Pharm. Res*, 4(6), 3319-3323.

Ghadeer, J. (2009). *Study the Kinetic Characterizations of New Photo-Stabilizer(4-Stearoylresorcinol) Production Using Batch Reactor*. (M. Sc. Thesis), Chemical Engineering Department, Basrah University.

Gueu, S., Yao, B., Adouby, K., & Ado, G. (2006). Heavy Metals Removal in Aqueous Solution by Activated Carbons Prepared from Coconut Shell and Seed Shell of the Plam Tree. *Journal of Applied Sciences*, 6(13), 2789-2793. <http://dx.doi.org/10.3923/jas.2006.2789.2793>

Hameed, B. H., Ahmad, A. A., & Aziz, N. (2011). Adsorption of Reactive Dye on Palm-Oil Industry Waste: Equilibrium, Kinetic and Thermodynamic Studies. *Journal of Desalination*, 247, 551-560. <http://dx.doi.org/10.1016/j.desal.2008.08.005>

Itodo, A. U., Abdulrahman, F. W., Hassan, L. G., Maigandi, S. A., & Itodo, H. U. (2010). Intraparticle Diffusion and Intraparticulate Diffusivities of Herbicide on Derived Activated Carbon. *Researcher*, 2(2), 74-86.

Itodo, H. U., & Itodo, A. U. (2010). Surface Coverage and Adsorption Study of Dye Uptake by Derived Acid and Base Treated Mango Seed Shells. *Journal Chem. Pharm. Res.*, 2(3), 673-683.

Jaguaribe, E. F., Medeiros, L. L., Barreto, M. C. S., & Araujo, L. P. (2000). The Performance of Activated Carbons From Sugarcane Bagasse, Babassu, and Coconut Shells in Removing Residual Chlorine. *Brazilian Journal of Chemical Engineering*, 22(1), 41-47. <http://dx.doi.org/10.1590/S0104-66322005000100005>

John, W. M. P. (1998). *Statistical Design and Analysis of Experiments*. Society for Industrial and Applied Mathematics. <http://dx.doi.org/10.1137/1.9781611971149>

Juan, F. G., Encinar, J. M., Carmen, M., Sabio, E., Ramiro, A., Canito, J., & Ganán, J. (2005). Preparation of activated carbons from used tyres by gasification with steam and carbon dioxide. *Applied Surface Science*, 252, 5999-6004. <http://dx.doi.org/10.1016/j.apsusc.2005.11.029>

Kannan, & Veemaraj. (2009). Removal of Lead (II) Ions by Adsorption onto Bamboo Dust and Commercial Activated Carbons-A Comparative Study. *Journal of Chemistry*, 6(2), 247-256.

Lazic, Z. R. (2004). *Design of Experiments in Chemical Engineering*. Wiley-VCH, Germany. <http://dx.doi.org/10.1002/3527604162>

Mohammadi, S. Z., Karimi, M. A., Afzali, D., & Mansouri, F. (2010). Removal of Pb(II) from aqueous solutions using activated carbon from Sea-buckthorn stones by chemical activation. *Journal of Desalination*, 262, 86-93. <http://dx.doi.org/10.1016/j.desal.2010.05.048>

Mui, E. L. K., Cheung, W. H., Valix, M., & McKay, G. (2010). Mesoporous activated carbon from waste tyre rubber for dye removal from effluents. *Journal of Microporous and Mesoporous Materials*, 130, 287-294. <http://dx.doi.org/10.1016/j.micromeso.2009.11.022>

Nwabanne, J. T., & Igbokwe, P. K. (2012). Adsorption Performance of Packed Bed Column for the removal of Lead (II) using oil Palm Fibre. *International Journal of Applied Science and Technology*, 2(5), 106-115.

Philip, C. A., & Girgis, B. S. (1996). Adsorption Characteristics of Microporous Carbons from Apricot Stones Activated by Phosphoric Acid. *Journal of Chemical Technology and Biotechnology*, 67(3), 248-254. [http://dx.doi.org/10.1002/\(SICI\)1097-4660\(199611\)67:3<248::AID-JCTB557>3.0.CO;2-1](http://dx.doi.org/10.1002/(SICI)1097-4660(199611)67:3<248::AID-JCTB557>3.0.CO;2-1)

Prabakaran, R., & Arivoli, S. (2012). Equilibrium Isotherm, Kinetic and Thermodynamic studies of Rhodamine B Adsorption using Thespesiapopulnea Bark. *Journal of Chem. Pharm. Res.*, 4(10), 4550-4557.

Rosa, G. D., Reynel-Avila, H. E., Bonilla-Petriciolet, A., Cano-Rodriguez, I., Velasco-Santos, C., & Martinez-Hernandez, A. L. (2008). Recycling Poultry Feathers for Pb Removal from Wastewater: Kinetic and Equilibrium Studies. *World Academy of Science, Engineering and Technology*, 47, 394-402.

Satya, S. P. M., Ahmed, J., & Krishnaiah, K. (1997). Production of Activated Carbon from Coconut Shell Char in a Fluidized Bed Reactor. *Industrial and Engineering Chemistry Research Journal*, 36(9), 3625-3630. <http://dx.doi.org/10.1021/ie970190v>

Shanthi, T., & Selvarajan, V. M. (2012). Removal of Cu (II) Ions from Aqueous Solution by Carbon Prepared from Henna Leaves, Pungam Bark and CAC. *Journal Chem. Pharm. Res.*, 4(9), 4296-4306.

Song, X., Zhang, Y., Yan, C., Jiang, W., & Chang, C. (2012). The Langmuir monolayer adsorption model of organic matter into effective pores in activated carbon. *Journal of Colloid and Interface Science*, 389, 213-219. <http://dx.doi.org/10.1016/j.jcis.2012.08.060>

Souag, R., Touaibia, D., Benayada, B., & Boucenna, A. (2009). Adsorption of Heavy Metals (Cd, Zn and Pb) from Water Using Keratin Powder Prepared from Algerien Sheep Hoofs. *European Journal of Scientific Research*, 35(3), 416-425.

Surchi, K. M. S. (2011). Agricultural Wastes as Low Cost Adsorbents for Pb Removal: Kinetics, Equilibrium and Thermodynamics. *International Journal of Chemistry*, 3(3), 103-112. <http://dx.doi.org/10.5539/ijc.v3n3p103>

Tan, I. A. W., Ahmad, A. L., & Hameed, B. H. (2007). Adsorption of basic dye using activated carbon prepared from oil palm shell: batch and fixed bed studies. *Journal of Desalination*, 225, 13-28. <http://dx.doi.org/10.1016/j.desal.2007.07.005>

Tan, I. A. W., Ahmad, A. L., & Hameed, B. H. (2007). Preparation of activated carbon from coconut husk: Optimization study on removal of 2,4,6-trichlorophenol using response surface methodology. *Journal of Hazardous Materials*, 153, 709-717. <http://dx.doi.org/10.1016/j.jhazmat.2007.09.014>

Teng, H., Lin, Y., & Hsu, L. (2000). Production of Activated Carbons from Pyrolysis of Waste Tires Impregnated with Potassium Hydroxide. *Journal of the Air & Waste Management Association*, 50, 1940-1946. <http://dx.doi.org/10.1080/10473289.2000.10464221>

Appendix

Appendix A.1. The experimental result of adsorption of heavy metal ions on the RHAC

No.	PH	T Ć	Dose	Conc.	rpm	Cu ²⁺ R.P. %	Fe ³⁺ R.P. %	Pb ²⁺ R.P. %	Zn ²⁺ R.P. %
1	9	50	4	50	750	81.23938	88.10662	83.82919	67.9962
2	5	50	4	50	750	84.54624	85.33153	85.56493	52.9737
3	9	22	4	50	750	74.00122	81.08255	79.39755	56.61451
4	5	22	4	50	750	76.60166	79.10268	81.62985	42.66601
5	9	50	2	50	750	69.63233	74.32959	72.50283	56.61451
6	5	50	2	50	750	73.71796	71.47762	74.83404	42.66601
7	9	22	2	50	750	64.84563	64.78792	68.94422	49.15597
8	5	22	2	50	750	67.21829	63.03204	70.13866	38.46659
9	9	50	4	21	750	90.37154	93.31052	89.2343	80.17145
10	5	50	4	21	750	92.9487	91.29079	93.56186	73.17979
11	9	22	4	21	750	85.91279	84.86117	84.87837	76.66654
12	5	22	4	21	750	87.62928	83.32352	91.40162	70.96144
13	9	50	2	21	750	80.79923	86.42614	84.87837	70.3359
14	5	50	2	21	750	82.03279	81.81263	87.05988	61.41432
15	9	22	2	21	750	78.91772	80.32793	81.59281	64.78172
16	5	22	2	21	750	79.54887	76.02539	85.97001	54.57449
17	9	50	4	50	350	83.60205	88.35708	84.04731	68.70761
18	5	50	4	50	350	85.7452	85.58079	85.13297	50.95187
19	9	22	4	50	350	73.71796	80.78995	79.1725	57.98543
20	5	22	4	50	350	76.22091	78.45418	82.07236	41.31281
21	9	50	2	50	350	70.99306	74.90697	71.32487	58.76964
22	5	50	2	50	350	74.11563	72.93589	74.6024	41.76894
23	9	22	2	50	350	65.5934	64.54013	69.90043	50.60096
24	5	22	2	50	350	68.79341	61.61917	71.56112	39.4365
25	9	50	4	21	350	91.42334	93.48079	89.12575	80.4263
26	5	50	4	21	350	92.69899	91.29079	93.56186	75.56583
27	9	22	4	21	350	84.44523	81.81263	85.53357	77.66965
28	5	22	4	21	350	87.91037	86.42614	90.31885	72.31815
29	9	50	2	21	350	81.41817	86.42614	85.97001	73.86935
30	5	50	2	21	350	81.72603	82.56477	88.14798	61.76971

31	9	22	2	21	350	77.64412	81.06704	82.68977	66.5229
32	5	22	2	21	350	79.86298	76.30535	84.87837	55.21746
33	2.243	36	3	35.5	550	65.16427	53.23355	82.7634	33.13241
34	11.76	36	3	35.5	550	47.34202	65.34776	74.23148	56.28086
35	7	2.702	3	35.5	550	84.75804	84.42365	83.76516	67.8024
36	7	69.3	3	35.5	550	93.10076	91.87511	90.64515	78.72574
37	7	36	0.6216	35.5	550	51.82756	58.27698	41.58728	30.38296
38	7	36	5.3784	35.5	550	97.12532	99.60192	93.83991	91.40932
39	7	36	3	1.013	550	100	100	100	98.8141
40	7	36	3	69.99	550	87.49923	85.7259	81.19688	64.50927
41	7	36	3	35.5	74.32	78.67269	79.63733	73.18559	57.88295
42	7	36	3	35.5	1026	93.75402	88.79539	89.35254	71.35046
43	7	36	3	35.5	550	96.60561	95.66103	89.35254	73.14867
44	7	36	3	35.5	550	96.06727	95.66103	90.96699	74.06098
45	7	36	3	35.5	550	97.94627	93.81068	89.9999	70.46211
46	7	36	3	35.5	550	96.95424	95.98851	88.05152	73.14867
47	7	36	3	35.5	550	97.46075	91.87511	91.28831	75.91122
48	7	36	3	35.5	550	96.2487	92.67749	88.05152	75.91122
49	7	36	3	35.5	550	97.21003	93.52026	84.43038	74.98215
50	7	36	3	35.5	550	96.95424	94.71204	89.9999	72.24541
51	7	36	3	35.5	550	95.32283	92.95377	87.39786	74.98215
52	7	36	3	35.5	550	95.03638	93.81068	88.37756	74.06098

Appendix A.2. F_T Values

α	f_2	f_1 [for greater variance]																	f_2		
		1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60		120	∞
0.1		2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63	
0.05	19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	19
0.01		8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49	
0.1		2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61	
0.05	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	20
0.01		8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42	
0.1		2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59	
0.05	21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	21
0.01		8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36	
0.1		2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.85	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57	
0.05	22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	22
0.01		7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31	
0.1		2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.85	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55	
0.05	23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86	1.81	1.76	23
0.01		7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.25	
0.1		2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53	
0.05	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.35	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	24
0.01		7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21	
0.1		2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52	
0.05	25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	25
0.01		7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17	
0.1		2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50	
0.05	26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	26
0.01		7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.82	2.66	2.58	2.50	2.42	2.33	2.23	2.13	

Appendix B. The calculation and results of models were determined by Minitab program

Central Composite Design

Factors: 5 Replicates: 1
 Base runs: 52 Total runs: 52
 Base blocks: 1 Total blocks: 1

Two-level factorial: Full factorial

Cube points: 32
 Center points in cube: 10
 Axial points: 10
 Center points in axial: 0

Alpha: 2.37841

Design Table

Run	Blk	A	B	C	D	E
1	1	-1.00000	-1.00000	-1.00000	-1.00000	-1.00000
2	1	1.00000	-1.00000	-1.00000	-1.00000	-1.00000
3	1	-1.00000	1.00000	-1.00000	-1.00000	-1.00000
4	1	1.00000	1.00000	-1.00000	-1.00000	-1.00000
5	1	-1.00000	-1.00000	1.00000	-1.00000	-1.00000
6	1	1.00000	-1.00000	1.00000	-1.00000	-1.00000
7	1	-1.00000	1.00000	1.00000	-1.00000	-1.00000
8	1	1.00000	1.00000	1.00000	-1.00000	-1.00000
9	1	-1.00000	-1.00000	-1.00000	1.00000	-1.00000
10	1	1.00000	-1.00000	-1.00000	1.00000	-1.00000
11	1	-1.00000	1.00000	-1.00000	1.00000	-1.00000
12	1	1.00000	1.00000	-1.00000	1.00000	-1.00000
13	1	-1.00000	-1.00000	1.00000	1.00000	-1.00000
14	1	1.00000	-1.00000	1.00000	1.00000	-1.00000
15	1	-1.00000	1.00000	1.00000	1.00000	-1.00000
16	1	1.00000	1.00000	1.00000	1.00000	-1.00000
17	1	-1.00000	-1.00000	-1.00000	-1.00000	1.00000
18	1	1.00000	-1.00000	-1.00000	-1.00000	1.00000
19	1	-1.00000	1.00000	-1.00000	-1.00000	1.00000
20	1	1.00000	1.00000	-1.00000	-1.00000	1.00000
21	1	-1.00000	-1.00000	1.00000	-1.00000	1.00000
22	1	1.00000	-1.00000	1.00000	-1.00000	1.00000
23	1	-1.00000	1.00000	1.00000	-1.00000	1.00000
24	1	1.00000	1.00000	1.00000	-1.00000	1.00000
25	1	-1.00000	-1.00000	-1.00000	1.00000	1.00000
26	1	1.00000	-1.00000	-1.00000	1.00000	1.00000
27	1	-1.00000	1.00000	-1.00000	1.00000	1.00000
28	1	1.00000	1.00000	-1.00000	1.00000	1.00000
29	1	-1.00000	-1.00000	1.00000	1.00000	1.00000
30	1	1.00000	-1.00000	1.00000	1.00000	1.00000
31	1	-1.00000	1.00000	1.00000	1.00000	1.00000
32	1	1.00000	1.00000	1.00000	1.00000	1.00000
33	1	-2.37841	0.00000	0.00000	0.00000	0.00000
34	1	2.37841	0.00000	0.00000	0.00000	0.00000
35	1	0.00000	-2.37841	0.00000	0.00000	0.00000
36	1	0.00000	2.37841	0.00000	0.00000	0.00000
37	1	0.00000	0.00000	-2.37841	0.00000	0.00000
38	1	0.00000	0.00000	2.37841	0.00000	0.00000
39	1	0.00000	0.00000	0.00000	-2.37841	0.00000
40	1	0.00000	0.00000	0.00000	2.37841	0.00000
41	1	0.00000	0.00000	0.00000	0.00000	-2.37841
42	1	0.00000	0.00000	0.00000	0.00000	2.37841
43	1	0.00000	0.00000	0.00000	0.00000	0.00000
44	1	0.00000	0.00000	0.00000	0.00000	0.00000
45	1	0.00000	0.00000	0.00000	0.00000	0.00000
46	1	0.00000	0.00000	0.00000	0.00000	0.00000
47	1	0.00000	0.00000	0.00000	0.00000	0.00000
48	1	0.00000	0.00000	0.00000	0.00000	0.00000
49	1	0.00000	0.00000	0.00000	0.00000	0.00000
50	1	0.00000	0.00000	0.00000	0.00000	0.00000
51	1	0.00000	0.00000	0.00000	0.00000	0.00000
52	1	0.00000	0.00000	0.00000	0.00000	0.00000

Adsorption of Cu^{2+} in batch presses

Response Surface Regression: response versus X1, X2, X3, X4, X5

The analysis was done using coded units.

Estimated Regression Coefficients for response

Term	Coef	SE Coef	T	P
Constant	96.3394	1.2160	79.229	0.000
X1	-1.8273	0.5879	-3.108	0.004
X2	2.4932	0.5879	4.241	0.000
X3	6.0002	0.5879	10.207	0.000
X4	-4.4891	0.5879	-7.636	0.000
X5	0.6908	0.5879	1.175	0.249
X1*X1	-7.4109	0.5057	-14.654	0.000
X2*X2	-1.6345	0.5057	-3.232	0.003
X3*X3	-4.1894	0.5057	-8.284	0.000
X4*X4	-0.7824	0.5057	-1.547	0.132
X5*X5	-2.1146	0.5057	-4.181	0.000
X1*X2	0.0205	0.6839	0.030	0.976
X1*X3	-0.0755	0.6839	-0.110	0.913
X1*X4	-0.3096	0.6839	-0.453	0.654
X1*X5	-0.0090	0.6839	-0.013	0.990
X2*X3	0.7539	0.6839	1.102	0.279
X2*X4	0.7829	0.6839	1.145	0.261
X2*X5	-0.2163	0.6839	-0.316	0.754
X3*X4	0.2930	0.6839	0.428	0.671
X3*X5	0.0288	0.6839	0.042	0.967
X4*X5	-0.2503	0.6839	-0.366	0.717

S = 3.86885 PRESS = 2132.64
 R-Sq = 93.77% R-Sq(pred) = 71.38% R-Sq(adj) = 89.76%

Analysis of Variance for response

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	20	6987.36	6987.36	349.37	23.34	0.000
Linear	5	2866.80	2866.80	573.36	38.31	0.000
X1	1	144.63	144.63	144.63	9.66	0.004
X2	1	269.24	269.24	269.24	17.99	0.000
X3	1	1559.41	1559.41	1559.41	104.18	0.000
X4	1	872.84	872.84	872.84	58.31	0.000
X5	1	20.67	20.67	20.67	1.38	0.249
Square	5	4073.22	4073.22	814.64	54.43	0.000
X1*X1	1	2784.90	3214.33	3214.33	214.75	0.000
X2*X2	1	74.41	156.35	156.35	10.45	0.003
X3*X3	1	930.88	1027.21	1027.21	68.63	0.000
X4*X4	1	21.33	35.82	35.82	2.39	0.132
X5*X5	1	261.70	261.70	261.70	17.48	0.000
Interaction	10	47.34	47.34	4.73	0.32	0.971
X1*X2	1	0.01	0.01	0.01	0.00	0.976
X1*X3	1	0.18	0.18	0.18	0.01	0.913
X1*X4	1	3.07	3.07	3.07	0.20	0.654
X1*X5	1	0.00	0.00	0.00	0.00	0.990
X2*X3	1	18.19	18.19	18.19	1.22	0.279
X2*X4	1	19.61	19.61	19.61	1.31	0.261
X2*X5	1	1.50	1.50	1.50	0.10	0.754
X3*X4	1	2.75	2.75	2.75	0.18	0.671
X3*X5	1	0.03	0.03	0.03	0.00	0.967
X4*X5	1	2.01	2.01	2.01	0.13	0.717
Residual Error	31	464.01	464.01	14.97		
Lack-of-Fit	22	456.35	456.35	20.74	24.38	0.000
Pure Error	9	7.66	7.66	0.85		
Total	51	7451.37				

Unusual Observations for response

Obs	StdOrder	response	Fit	SE Fit	Residual	St Resid
33	33	65.164	58.763	2.895	6.401	2.49 R
37	37	51.828	58.369	2.895	-6.542	-2.55 R
38	38	97.125	86.912	2.895	10.214	3.98 R
40	40	87.499	81.237	2.895	6.262	2.44 R
42	42	93.754	86.020	2.895	7.734	3.01 R

R denotes an observation with a large standardized residual.

Adsorption of Fe^{3+} in batch presses

Response Surface Regression: Response versus X1, X2, X3, X4, X5

The analysis was done using coded units.

Estimated Regression Coefficients for Response

Term	Coef	SE Coef	T	P
Constant	93.8683	0.9561	98.180	0.000
X1	1.5434	0.4622	3.339	0.002
X2	3.2736	0.4622	7.083	0.000
X3	6.2868	0.4622	13.601	0.000
X4	-4.0695	0.4622	-8.804	0.000
X5	0.4583	0.4622	0.992	0.329
X1*X1	-6.3800	0.3976	-16.045	0.000
X2*X2	-1.2784	0.3976	-3.215	0.003
X3*X3	-2.9065	0.3976	-7.310	0.000
X4*X4	-0.4452	0.3976	-1.120	0.271
X5*X5	-1.9737	0.3976	-4.964	0.000
X1*X2	0.2524	0.5378	0.469	0.642
X1*X3	-0.5012	0.5378	-0.932	0.359
X1*X4	0.0217	0.5378	0.040	0.968
X1*X5	0.1760	0.5378	0.327	0.746
X2*X3	-0.0712	0.5378	-0.132	0.896
X2*X4	0.3489	0.5378	0.649	0.521
X2*X5	-0.1558	0.5378	-0.290	0.774
X3*X4	2.0105	0.5378	3.739	0.001
X3*X5	0.0739	0.5378	0.137	0.892
X4*X5	0.0644	0.5378	0.120	0.905

S = 3.04198 PRESS = 1227.20
 R-Sq = 95.42% R-Sq(pred) = 80.41% R-Sq(adj) = 92.46%

Analysis of Variance for Response

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	20	5976.07	5976.07	298.80	32.29	0.000
Linear	5	3005.72	3005.72	601.14	64.96	0.000

Analysis of Variance for Response

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	20	5976.07	5976.07	298.80	32.29	0.000
Linear	5	3005.72	3005.72	601.14	64.96	0.000
X1	1	103.18	103.18	103.18	11.15	0.002
X2	1	464.18	464.18	464.18	50.16	0.000
X3	1	1711.93	1711.93	1711.93	185.00	0.000
X4	1	717.32	717.32	717.32	77.52	0.000
X5	1	9.10	9.10	9.10	0.98	0.329
Square	5	2824.79	2824.79	564.96	61.05	0.000
X1*X1	1	2107.50	2382.26	2382.26	257.44	0.000
X2*X2	1	47.19	95.65	95.65	10.34	0.003
X3*X3	1	437.61	494.43	494.43	53.43	0.000
X4*X4	1	4.50	11.60	11.60	1.25	0.271
X5*X5	1	227.99	227.99	227.99	24.64	0.000
Interaction	10	145.57	145.57	14.56	1.57	0.161
X1*X2	1	2.04	2.04	2.04	0.22	0.642
X1*X3	1	8.04	8.04	8.04	0.87	0.359
X1*X4	1	0.02	0.02	0.02	0.00	0.968
X1*X5	1	0.99	0.99	0.99	0.11	0.746
X2*X3	1	0.16	0.16	0.16	0.02	0.896
X2*X4	1	3.89	3.89	3.89	0.42	0.521
X2*X5	1	0.78	0.78	0.78	0.08	0.774
X3*X4	1	129.34	129.34	129.34	13.98	0.001
X3*X5	1	0.17	0.17	0.17	0.02	0.892
X4*X5	1	0.13	0.13	0.13	0.01	0.905
Residual Error	31	286.86	286.86	9.25		
Lack-of-Fit	22	269.27	269.27	12.24	6.26	0.004
Pure Error	9	17.59	17.59	1.95		
Total	51	6262.94				

Unusual Observations for Response

Obs	StdOrder	Response	Fit	SE Fit	Residual	St Resid
6	6	81.813	86.345	2.041	-4.533	-2.01 R
35	35	84.424	78.850	2.276	5.573	2.76 R
37	37	58.277	62.474	2.276	-4.197	-2.08 R
38	38	99.602	92.379	2.276	7.223	3.58 R
40	40	85.726	81.671	2.276	4.055	2.01 R
42	42	88.795	83.794	2.276	5.002	2.48 R

R denotes an observation with a large standardized residual.

Estimated Regression Coefficients for response using data in unco

Term	Coef
Constant	-65.9923
X1	25.5027
X2	0.517412
X3	28.6660
X4	-0.122617
X5	0.0671769
X1*X1	-1.85272
X2*X2	-0.00833916
X3*X3	-4.18942
X4*X4	-0.00372116
X5*X5	-5.28652E-05
X1*X2	0.000731314
X1*X3	-0.0377430
X1*X4	-0.0106772
X1*X5	-2.24801E-05
X2*X3	0.0538510
X2*X4	0.00385651
X2*X5	-7.72475E-05
X3*X4	0.0202038
X3*X5	0.000143868
X4*X5	-8.63207E-05

Adsorption of Pb⁺² in batch presses

Response Surface Regression: Response versus X1, X2, X3,

The analysis was done using coded units.

Estimated Regression Coefficients for Response

Term	Coef	SE Coef	T	P
Constant	88.8274	1.3666	64.997	0.000
X1	-1.5632	0.6607	-2.366	0.024
X2	1.6083	0.6607	2.434	0.021
X3	5.7198	0.6607	8.657	0.000
X4	-4.8223	0.6607	-7.299	0.000
X5	0.8272	0.6607	1.252	0.220
X1*X1	-1.7780	0.5684	-3.128	0.004
X2*X2	-0.2387	0.5684	-0.420	0.677
X3*X3	-3.6843	0.5684	-6.482	0.000
X4*X4	0.3612	0.5684	0.635	0.530
X5*X5	-1.2881	0.5684	-2.266	0.031
X1*X2	0.1346	0.7687	0.175	0.862
X1*X3	-0.2699	0.7687	-0.351	0.728
X1*X4	0.4556	0.7687	0.593	0.558
X1*X5	-0.0747	0.7687	-0.097	0.923
X2*X3	0.1878	0.7687	0.244	0.809
X2*X4	0.1483	0.7687	0.193	0.848
X2*X5	0.0539	0.7687	0.070	0.945
X3*X4	1.5815	0.7687	2.057	0.048
X3*X5	0.1152	0.7687	0.150	0.882
X4*X5	0.0211	0.7687	0.027	0.978

S = 4.34826 PRESS = 2604.31
 R-Sq = 86.56% R-Sq(pred) = 40.27% R-Sq(adj) = 77.88%

Estimated Regression Coefficients for Response using data in uncoded un

Term	Coef
Constant	-45.2262
X1	23.2603
X2	0.625204
X3	20.5380
X4	-0.625608
X5	0.0535949
X1*X1	-1.59499
X2*X2	-0.00652259
X3*X3	-2.90653
X4*X4	-0.00211738
X5*X5	-4.93424E-05
X1*X2	0.00901583
X1*X3	-0.250596
X1*X4	0.000747776
X1*X5	0.000440045
X2*X3	-0.00508635
X2*X4	0.00171861
X2*X5	-5.56518E-05
X3*X4	0.138653
X3*X5	0.000369255
X4*X5	2.22194E-05

Adsorption of Zn⁺² in batch presses

Response Surface Regression: Response versus X1, X2, X3, X4, X5

The analysis was done using coded units.

Estimated Regression Coefficients for Response

Term	Coef	SE Coef	T	P
Constant	73.2958	1.8272	40.115	0.000
X1	5.4648	0.8833	6.187	0.000
X2	2.9600	0.8833	3.351	0.002
X3	7.0497	0.8833	7.981	0.000
X4	-8.7813	0.8833	-9.941	0.000
X5	0.4243	0.8833	0.480	0.634
X1*X1	-5.8553	0.7599	-7.705	0.000
X2*X2	-0.8070	0.7599	-1.062	0.296
X3*X3	-2.9934	0.7599	-3.939	0.000
X4*X4	0.6775	0.7599	0.892	0.380
X5*X5	-2.3357	0.7599	-3.074	0.004
X1*X2	0.3611	1.0277	0.351	0.728
X1*X3	-0.2822	1.0277	-0.275	0.785
X1*X4	1.5862	1.0277	1.544	0.133
X1*X5	-0.3368	1.0277	-0.328	0.745
X2*X3	0.1665	1.0277	0.162	0.872
X2*X4	0.8184	1.0277	0.796	0.432
X2*X5	0.0219	1.0277	0.021	0.983
X3*X4	-1.1483	1.0277	-1.117	0.272
X3*X5	0.1949	1.0277	0.190	0.851
X4*X5	0.2779	1.0277	0.270	0.789

S = 5.81350 PRESS = 4679.38
 R-Sq = 90.52% R-Sq(pred) = 57.64% R-Sq(adj) = 84.40%

Analysis of Variance for Response

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	20	3774.03	3774.03	188.70	9.98	0.000
Linear	5	2671.79	2671.79	534.36	28.26	0.000
X1	1	105.84	105.84	105.84	5.60	0.024
X2	1	112.04	112.04	112.04	5.93	0.021
X3	1	1417.05	1417.05	1417.05	74.95	0.000
X4	1	1007.22	1007.22	1007.22	53.27	0.000
X5	1	29.64	29.64	29.64	1.57	0.220
Square	5	1010.10	1010.10	202.02	10.68	0.000
X1*X1	1	125.62	185.02	185.02	9.79	0.004
X2*X2	1	0.57	3.33	3.33	0.18	0.677
X3*X3	1	773.71	794.46	794.46	42.02	0.000
X4*X4	1	13.10	7.63	7.63	0.40	0.530
X5*X5	1	97.10	97.10	97.10	5.14	0.031
Interaction	10	92.14	92.14	9.21	0.49	0.885
X1*X2	1	0.58	0.58	0.58	0.03	0.862
X1*X3	1	2.33	2.33	2.33	0.12	0.728
X1*X4	1	6.64	6.64	6.64	0.35	0.558
X1*X5	1	0.18	0.18	0.18	0.01	0.923
X2*X3	1	1.13	1.13	1.13	0.06	0.809
X2*X4	1	0.70	0.70	0.70	0.04	0.848
X2*X5	1	0.09	0.09	0.09	0.00	0.945
X3*X4	1	80.04	80.04	80.04	4.23	0.048
X3*X5	1	0.42	0.42	0.42	0.02	0.882
X4*X5	1	0.01	0.01	0.01	0.00	0.978
Residual Error	31	586.13	586.13	18.91		
Lack-of-Fit	22	549.70	549.70	24.99	6.17	0.004
Pure Error	9	36.43	36.43	4.05		
Total	51	4360.16				

Unusual Observations for Response

Obs	StdOrder	Response	Fit	SE Fit	Residual	St Resid
37	37	41.587	54.382	3.254	-12.794	-4.44 R
38	38	93.840	81.590	3.254	12.250	4.25 R
41	41	73.186	79.574	3.254	-6.388	-2.21 R
42	42	89.353	83.509	3.254	5.844	2.03 R

R denotes an observation with a large standardized residual.

Estimated Regression Coefficients for Response using data in uncoded unit

Term	Coef
Constant	35.4757
X1	5.21821
X2	0.0921399
X3	24.0988
X4	-0.922029
X5	0.0381852
X1*X1	-0.444504
X2*X2	-0.00121782
X3*X3	-3.68435
X4*X4	0.00171777
X5*X5	-3.22014E-05
X1*X2	0.00480844
X1*X3	-0.134947
X1*X4	0.0157113
X1*X5	-1.86838E-04
X2*X3	0.0134119
X2*X4	0.000730485
X2*X5	1.92648E-05
X3*X4	0.109071
X3*X5	0.000576037
X4*X5	7.28692E-06

Analysis of Variance for Response

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	20	9998.3	9998.29	499.91	14.79	0.000
Linear	5	7173.4	7173.39	1434.68	42.45	0.000
X1	1	1293.5	1293.53	1293.53	38.27	0.000
X2	1	379.5	379.51	379.51	11.23	0.002
X3	1	2152.6	2152.61	2152.61	63.69	0.000
X4	1	3339.9	3339.94	3339.94	98.82	0.000
X5	1	7.8	7.80	7.80	0.23	0.634
Square	5	2665.8	2665.82	533.16	15.78	0.000
X1*X1	1	1802.0	2006.55	2006.55	59.37	0.000
X2*X2	1	13.0	38.12	38.12	1.13	0.296
X3*X3	1	486.2	524.41	524.41	15.52	0.000
X4*X4	1	45.4	26.86	26.86	0.79	0.380
X5*X5	1	319.3	319.28	319.28	9.45	0.004
Interaction	10	159.1	159.08	15.91	0.47	0.896
X1*X2	1	4.2	4.17	4.17	0.12	0.728
X1*X3	1	2.5	2.55	2.55	0.08	0.785
X1*X4	1	80.5	80.52	80.52	2.38	0.133
X1*X5	1	3.6	3.63	3.63	0.11	0.745
X2*X3	1	0.9	0.89	0.89	0.03	0.872
X2*X4	1	21.4	21.43	21.43	0.63	0.432
X2*X5	1	0.0	0.02	0.02	0.00	0.983
X3*X4	1	42.2	42.19	42.19	1.25	0.272
X3*X5	1	1.2	1.22	1.22	0.04	0.851
X4*X5	1	2.5	2.47	2.47	0.07	0.789
Residual Error	31	1047.7	1047.70	33.80		
Lack-of-Fit	22	1021.5	1021.53	46.43	15.97	0.000
Pure Error	9	26.2	26.17	2.91		
Total	51	11046.0				

Unusual Observations for Response

Obs	StdOrder	Response	Fit	SE Fit	Residual	St Resid
37	37	30.383	39.596	4.351	-9.213	-2.39 R
38	38	91.409	73.130	4.351	18.280	4.74 R
40	40	64.509	56.243	4.351	8.266	2.14 R
42	42	71.350	61.092	4.351	10.258	2.66 R

Unusual Observations for Response

Obs	StdOrder	Response	Fit	SE Fit	Residual	St Resid
37	37	30.383	39.596	4.351	-9.213	-2.39 R
38	38	91.409	73.130	4.351	18.280	4.74 R
40	40	64.509	56.243	4.351	8.266	2.14 R
42	42	71.350	61.092	4.351	10.258	2.66 R

R denotes an observation with a large standardized residual.

Estimated Regression Coefficients for Response using data in uncoded unit

Term	Coef
Constant	-59.5905
X1	21.7061
X2	0.234516
X3	27.8446
X4	-1.17755
X5	0.0656383
X1*X1	-1.46382
X2*X2	-0.00411741
X3*X3	-2.99337
X4*X4	0.00322231
X5*X5	-5.83916E-05
X1*X2	0.0128980
X1*X3	-0.141076
X1*X4	0.0546980
X1*X5	-8.41884E-04
X2*X3	0.0118893
X2*X4	0.00403162
X2*X5	7.80374E-06
X3*X4	-0.0791903
X3*X5	0.000974679
X4*X5	9.58348E-05

Copyright Disclaimer

Copyright reserved by the author(s).

This article is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by/3.0/>).