

How Can We Interpret the Estimates of the Full BEKK Model with Asymmetry? The Case of French and German Stock Returns

Chikashi Tsuji

Professor, Faculty of Economics, Chuo University

742-1 Higashinakano, Hachioji-shi, Tokyo 192-0393, Japan

E-mail: mail_sec_low@minos.ocn.ne.jp

Received: September 23, 2017 Accepted: October 19, 2017 Published: October 27, 2017

doi:10.5296/ber.v7i2.12071

URL: <http://dx.doi.org/10.5296/ber.v7i2.12071>

Abstract

This study conducts careful interpretations of the model parameters from the full Baba-Engle-Kraft-Kroner (BEKK) model with asymmetric effects. This study also includes a case study, in which we interpret the full BEKK model parameter estimates from the empirical examinations using French and German stock index returns. More concretely, in this paper, we firstly examine the model formula and obtain general interpretations of the full BEKK model parameters. This shall be particularly helpful to understand not only the structure of the full BEKK model but also the mechanisms of similar multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) models. After the above general considerations, this study also interprets the case results, in which the full BEKK model is applied to French and German stock index returns. The concrete illustrations demonstrated in this case study shall be also very useful for future related research.

Keywords: French stock market, BEKK model, MGARCH model, German stock market

1. Introduction

In recent studies of asset pricing, multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) models are often employed to analyze financial time-series. We point out that, however, interpreting the results from applications of MGARCH models is not an easy work. It is considered that this is because these model structures are rather complicated and thus for applied or empirical researchers, obtaining the clear understanding of the detailed model structures and exact parameter meanings is not easy. However, in fact, in order to clearly understand the empirical results derived from these models, it is highly important to grasp how we can interpret the estimates from these econometric models.

Based on this viewpoint, this paper attempts careful interpretations as to the model parameters of an interesting model, the full Baba-Engle-Kraft-Kroner (BEKK) model with asymmetric effects (Baba, Engle, Kraft, and Kroner, 1990; Engle and Kroner, 1995). We also examine the parameter estimates obtained from a case study, which uses two European stock index returns. Attempting to derive clear interpretations of the model parameters is the objective of this article. As for the procedures more concretely, this paper firstly investigates the model formula, and then shows how we can generally interpret the model parameters. This shall be particularly helpful for understanding the structures of MGARCH models. This is a contribution of this paper. After general examinations, we also interpret the case results from the full BEKK model application to French and German stock index returns. The concrete illustrations presented via this case study shall be also very useful for future related research. This is another contribution of this article.

After this introduction, Section 2 conducts a review of related literature; Section 3 explains the data and variables we use for our case study; and Section 4 inspects the full BEKK model with asymmetry. After these, Section 5 derives the general interpretation of the model; Section 6 interprets our case results; and finally, in Section 7, we conclude this article.

2. Literature Review

In economics and finance, after the BEKK model was developed by Baba et al. (1990) and Engle and Kroner (1995), many empirical research used this MGARCH model. This section briefly reviews very recent related previous studies, which used the BEKK models. Gounopoulos et al. (2013) examined the linkages between stock returns and currency exposures of US, UK, and Japanese banks and insurance companies by using a BEKK model. Long et al. (2014) analyzed the conditional time-varying currency betas for five developed and six emerging financial markets by using a BEKK model.

Employing a BEKK model, Caporale et al. (2015) tested the impact of exchange rate uncertainty on net equity and net bond flows and on their dynamic linkages. Using BEKK and dynamic conditional correlation (DCC) models of Engle (2002), Zhou (2016) compared the hedge-ratio estimations for REIT futures. The objective of Olson et al. (2017) is to evaluate whether commodities have an effective function as a hedging tool for equity investors. Employing a BEKK model, they computed time-varying hedge ratios for the US equity index. Cardona et al. (2017) examined the volatility transmission between US and

Latin American stock markets by using a BEKK model. As the above recent literature review, many studies derived several time-varying information by using BEKK models; however, it is pointed out that interpreting the BEKK model parameter estimates is generally difficult.

3. Data and Variables

In this section, we document the data and variables for our case study. We examine two daily European stock index returns. First is the log percentage return series of the CAC 40 index in France, which we denote as LRCAC. Second is the log percentage return series of the DAX 30 index in Germany, which we denote as LRDAX. The sample period for our analyses is from August 4, 1987 to February 28, 2017; and all price data are from Thomson Reuters.

4. The Full BEKK Model with Asymmetry

We next explain the full BEKK model with asymmetric terms, which we inspect in this study. The model is specified as follows:

$$\mathbf{H}(t) = \mathbf{C}\mathbf{C}' + \mathbf{A}'\mathbf{u}(t-1)\mathbf{u}(t-1)'\mathbf{A} + \mathbf{B}'\mathbf{H}(t-1)\mathbf{B} + \mathbf{D}'\mathbf{v}(t-1)\mathbf{v}(t-1)'\mathbf{D}. \quad (1)$$

In our use of the model (1) for our case study, we employ only constant terms in the mean equations of LRCAC and LRDAX. In model (1), \mathbf{H} means the time-varying variance and covariance matrix as to LRCAC and LRDAX; and \mathbf{u} denotes the matrix of the residuals from the mean equations. Moreover, \mathbf{A} , \mathbf{B} , \mathbf{C} , and \mathbf{D} mean coefficient matrices and $\mathbf{v}(t-1) = \mathbf{u}(t-1) \circ I_{u < 0}(\mathbf{u}(t-1))$, where $\mathbf{u}(t-1) = [u_{1,t-1} \ u_{2,t-1}]'$, $\mathbf{v}(t-1) = [v_{1,t-1} \ v_{2,t-1}]'$, and \circ denotes the Hadamard product.

More specifically, the variance of the first asset return can be written as follows:

$$\begin{aligned} \sigma_{1,t}^2 = & C(1,1)^2 + A(1,1)^2 u_{1,t-1}^2 + 2A(1,1)A(2,1)u_{1,t-1}u_{2,t-1} + A(2,1)^2 u_{2,t-1}^2 \\ & + B(1,1)^2 \sigma_{1,t-1}^2 + 2B(1,1)B(2,1)\sigma_{12,t-1} + B(2,1)^2 \sigma_{2,t-1}^2 \\ & + D(1,1)^2 v_{1,t-1}^2 + 2D(1,1)D(2,1)v_{1,t-1}v_{2,t-1} + D(2,1)^2 v_{2,t-1}^2. \end{aligned} \quad (2)$$

In addition, the variance of the second asset return can be written as follows:

$$\begin{aligned} \sigma_{2,t}^2 = & C(2,1)^2 + C(2,2)^2 + A(1,2)^2 u_{1,t-1}^2 + 2A(1,2)A(2,2)u_{1,t-1}u_{2,t-1} + A(2,2)^2 u_{2,t-1}^2 \\ & + B(1,2)^2 \sigma_{1,t-1}^2 + 2B(1,2)B(2,2)\sigma_{12,t-1} + B(2,2)^2 \sigma_{2,t-1}^2 \\ & + D(1,2)^2 v_{1,t-1}^2 + 2D(1,2)D(2,2)v_{1,t-1}v_{2,t-1} + D(2,2)^2 v_{2,t-1}^2. \end{aligned} \quad (3)$$

Further, we can write the covariance of the first and second asset returns as follows:

$$\begin{aligned} \sigma_{12,t} = & C(1,1)C(2,1) + A(1,1)A(1,2)u_{1,t-1}^2 \\ & + (A(1,2)A(2,1) + A(1,1)A(2,2))u_{1,t-1}u_{2,t-1} + A(2,1)A(2,2)u_{2,t-1}^2 \\ & + B(1,1)B(1,2)\sigma_{1,t-1}^2 + (B(1,2)B(2,1) + B(1,1)B(2,2))\sigma_{12,t-1} + B(2,1)B(2,2)\sigma_{2,t-1}^2 \\ & + D(1,1)D(1,2)v_{1,t-1}^2 + (D(1,2)D(2,1) + D(1,1)D(2,2))v_{1,t-1}v_{2,t-1} + D(2,1)D(2,2)v_{2,t-1}^2. \end{aligned} \quad (4)$$

5. General Interpretations

5.1 Variances of the First and Second Asset Returns

This section attempts to interpret the model parameters generally. First, we interpret the parameters for the variance of the first (second) asset return. Inspecting the equation (2) ((3)), we find many squared coefficients; and these squared coefficients in equation (2) ((3)) always positively affect the first (second) asset return variance in the next period. Hence, we interpret the other terms and coefficients in equation (2) ((3)) below.

First, as to the effects of shocks in mean (return) equations, the coefficient $A(1,1)A(2,1)$ ($A(1,2)A(2,2)$) is generally difficult to interpret and the other terms have squared coefficients. Second, as to the effects of two asset return covariance, a positive $B(1,1)B(2,1)$ ($B(1,2)B(2,2)$) means that, assuming positive two asset return covariances, an increase of the two asset return covariance ups the first (second) asset return variance in the next period. Third, with regard to the asymmetric effects, a positive $D(1,1)D(2,1)$ ($D(1,2)D(2,2)$) means that when two asset returns have simultaneous negative shocks, these negative shocks shall increase the first (second) asset return variance in the next period.

5.2 Covariances of the First and Second Asset Returns

We next interpret the full BEKK model parameters for the two asset return covariance. First, from equation (4), with regard to the effects of shocks in mean equations, a positive $A(1,1)A(1,2)$ suggests a shock to the first asset return has a positive effect on the two asset return covariance in the next period; and a positive $A(2,1)A(2,2)$ means a shock to the second asset return positively affects the two asset return covariance in the next period. On the other hand, the coefficient $A(1,2)A(2,1)+A(1,1)A(2,2)$ is generally difficult to interpret.

Second, regarding the variance and covariance effects, a positive $B(1,1)B(1,2)$ means an increase of the first asset return variance has a positive effect on the two asset return covariance in the next period. Further, a positive $B(2,1)B(2,2)$ suggests an increase of the second asset return variance ups the two asset return covariance in the next period. Moreover, assuming that two asset return covariances are positive, a positive coefficient of $B(1,2)B(2,1)+B(1,1)B(2,2)$ indicates an increase of the two asset return covariance increases the two asset return covariance in the next period.

Moreover, regarding asymmetric effects, a positive $D(1,1)D(1,2)$ indicates a negative shock to the first asset return ups the two asset return covariance in the next period; and a positive $D(2,1)D(2,2)$ suggests a negative shock to the second asset return increases the two asset return covariance in the next period. Further, a positive coefficient of $D(1,2)D(2,1)+D(1,1)D(2,2)$ means that when two asset returns have simultaneous negative shocks, these negative shocks boost the two asset return covariance in the next period.

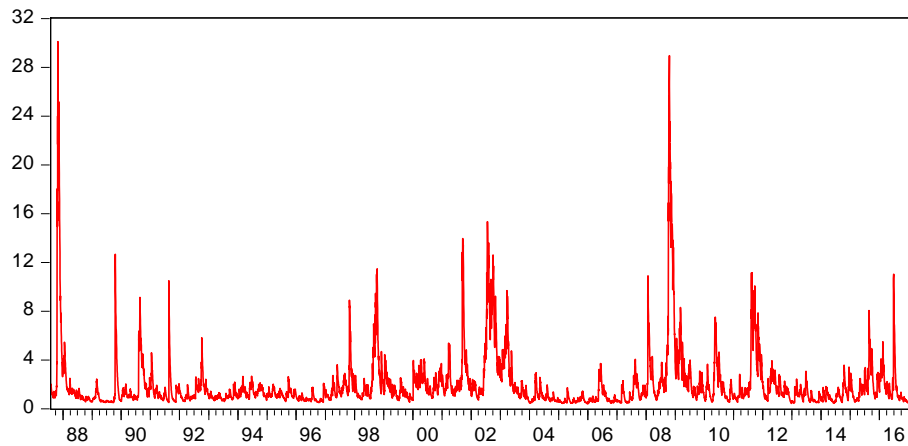
To sum up, as above, the parameter estimates from the full BEKK model with asymmetric effects are not easy to interpret in general. Thus, it is important to understand the effects and relations of the model parameters in order to clearly understand the empirical results from these kinds of econometric models.

Table 1. Estimation results of the full BEKK model with asymmetric terms for French and German stock index returns: For the period from August 4, 1987 to February 28, 2017

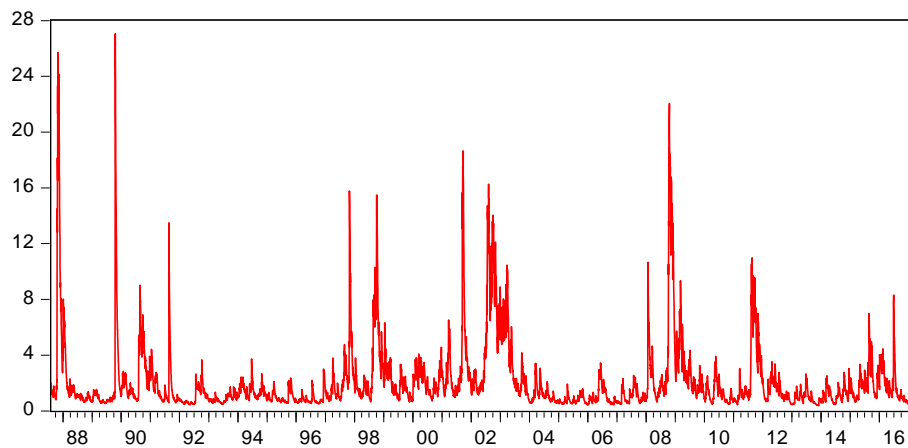
Mean equations				
	Estimates	Standard error	<i>t</i> -statistic	<i>p</i> -value
Const.(LRCAC)	0.0174***	0.0063	2.7646	0.0057
Const.(LRDAX)	0.0416***	0.0012	34.4154	0.0000
BEKK specifications				
	Estimates	Standard error	<i>t</i> -statistic	<i>p</i> -value
C(1,1)	0.1783***	0.0089	20.1412	0.0000
C(2,1)	0.1737***	0.0086	20.3079	0.0000
C(2,2)	0.0297***	0.0053	5.5480	0.0000
A(1,1)	0.1721***	0.0198	8.7093	0.0000
A(1,2)	-0.0607***	0.0175	-3.4633	0.0005
A(2,1)	-0.0917***	0.0185	-4.9475	0.0000
A(2,2)	0.1463***	0.0193	7.5733	0.0000
B(1,1)	0.9549***	0.0050	191.1719	0.0000
B(1,2)	-0.0064	0.0045	-1.4194	0.1558
B(2,1)	-0.0053	0.0044	-1.1980	0.2309
B(2,2)	0.9590***	0.0042	228.1226	0.0000
D(1,1)	0.2345***	0.0214	10.9399	0.0000
D(1,2)	-0.0013	0.0217	-0.0595	0.9526
D(2,1)	0.1366***	0.0210	6.5095	0.0000
D(2,2)	0.3494***	0.0220	15.8489	0.0000
LL	-19786.5165			

Notes: This table exhibits the results of maximum likelihood estimation in terms of the full BEKK model with asymmetry. The model estimation for French and German stock indices is conducted for the period from August 4, 1987 to February 28, 2017. In this table, $C(i,j)$ means the model parameter of the matrix C 's i -th row and j -th column; $A(i,j)$ means the model parameter of the matrix A 's i -th row and j -th column; $B(i,j)$ denotes the model parameter of the matrix B 's i -th row and j -th column; and $D(i,j)$ means the model parameter of the matrix D 's i -th row and j -th column, respectively. Further, *** denotes the statistical significance of the parameter estimates at the 1% level. LL is the log-likelihood value and Const. is the constant term.

Panel A. Evolution of French stock index return variances



Panel B. Evolution of German stock index return variances



Panel C. Evolution of French and German stock index return covariances

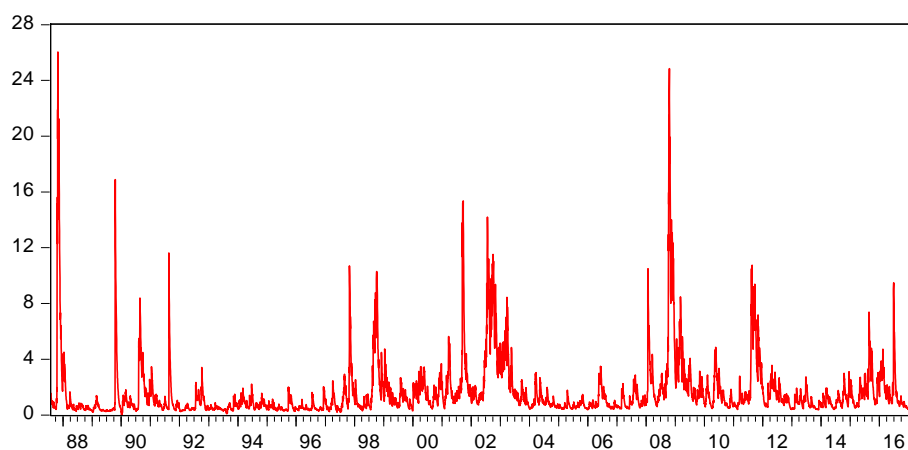


Figure 1. Dynamic time-series evolution of the variances and covariances of French and German equity index returns: Estimates derived from the full BEKK-MGARCH model with asymmetry for the period from August 4, 1987 to February 28, 2017.

6. Interpretations of the Case Results

In this section, we attempt to interpret our case study results from French and German stock index returns. First, Table 1 exhibits the estimation results of our full BEKK model with asymmetric terms. In Table 1, $C(i,j)$ means the model parameter of the matrix \mathbf{C} 's i -th row and j -th column; $A(i,j)$ means the model parameter of the matrix \mathbf{A} 's i -th row and j -th column; $B(i,j)$ denotes the model parameter of the matrix \mathbf{B} 's i -th row and j -th column; and $D(i,j)$ means the model parameter of the matrix \mathbf{D} 's i -th row and j -th column, respectively. These model parameters and four matrices are those in the full BEKK model (1). The results in Table 1 suggest that our full BEKK model with asymmetric terms is generally well estimated.

In addition, Figure 1 presents the evolution as to the time-varying variances and covariances of French and German stock index returns examined in our case study. Specifically, Panel A of Figure 1 shows the evolution of variances of French stock index returns; Panel B of this figure exhibits the evolution of variances of German stock index returns; and Panel C of this figure presents the evolution of covariances of French and German stock index returns. These time-series data are all depicted for the period from August 4, 1987 to February 28, 2017. As to these series, we attempt to interpret our full BEKK model parameter estimates below.

6.1 Variances of French Stock Index Returns

We begin by interpreting the model parameter estimates for the variance of the first asset return: the French stock index return. First, all the squared coefficients in equation (2) always positively affect the next day's French stock index return variance.

Second, as to the effects of two asset return covariance, the negative $B(1,1)B(2,1)$ of -0.0051 means that, assuming positive two asset return covariances, an increase of the two asset return covariance very weakly decreases the next day's French stock index return variance.

Third, as for the asymmetric effects, the positive $D(1,1)D(2,1)$ of 0.0320 means that when two asset returns have simultaneous negative shocks, these negative shocks shall increase the next day's French stock index return variance.

6.2 Variances of German Stock Index Returns

We next interpret the model parameter estimates for the variance of the second asset return: the German stock index return. First, all the squared coefficients in equation (3) always positively affect the next day's German stock index return variance.

Second, as to the effects of two asset return covariance, the negative coefficient $B(1,2)B(2,2)$ of -0.0061 means that, assuming positive two asset return covariances, an increase of the two asset return covariance very slightly decreases the next day's German stock index return variance.

Third, as for the asymmetric effects, the value for $D(1,2)D(2,2)$ of -0.0005 means that when two asset returns have simultaneous negative shocks, these negative shocks have little effect on the next day's German stock index return variance.

6.3 Covariances of French and German Stock Index Returns

Finally, we interpret the model parameter estimates for the two asset return covariance. First, with regard to the effects of shocks in mean equations, the negative $A(1,1)A(1,2)$ of -0.0104 suggests a shock to the French stock index return has a negative effect on the next day's two asset return covariance; and the negative $A(2,1)A(2,2)$ of -0.0134 means a shock to the German stock index return affects the next day's two asset return covariance negatively. On the other hand, the positive coefficient $A(1,2)A(2,1)+A(1,1)A(2,2)$ of 0.0307 is generally difficult to interpret.

Regarding the effects of variances and covariance, the negative $B(1,1)B(1,2)$ of -0.0061 means an increase of the French stock index return variance has a negative effect on the next day's two asset return covariance. Further, the negative $B(2,1)B(2,2)$ of -0.0051 indicates an increase of the German stock index return variance very weakly decreases the next day's two asset return covariance. Moreover, assuming positive two asset return covariances, the positive coefficient $B(1,2)B(2,1)+B(1,1)B(2,2)$ of 0.9158 indicates an increase of the two asset return covariance strongly ups the next day's two asset return covariance.

Moreover, as to the asymmetric effects, the value for $D(1,1)D(1,2)$ of -0.0003 suggests a negative shock to the French stock index return has little effect on the next day's two asset return covariance; and the positive $D(2,1)D(2,2)$ of 0.0477 indicates that a negative shock to the German stock index return increases the next day's two asset return covariance. Further, the positive $D(1,2)D(2,1)+D(1,1)D(2,2)$ of 0.0818 means that when two asset returns have simultaneous negative shocks, these negative shocks clearly up the next day's two asset return covariance. As this case study demonstrates, the parameter estimates from the full BEKK model with asymmetric effects are generally complicated to interpret. However, our case study explained above supplies a good illustration as to interpretations of the parameter estimates from the full BEKK model.

7. Conclusions

This study carefully interpreted the effects and relations of parameters of the full BEKK model with asymmetric effects. After general considerations, we also examined the case of French and German stock index returns by applying this full BEKK model. More specifically, we firstly inspected the model formula and obtained the general interpretations as to the effects and linkages of the model parameters. It is considered that this is particularly helpful to clearly understand the full BEKK model and similar MGARCH model structures as well. After the general considerations, as noted, we also attempted to interpret the case results, in which the full BEKK model was applied to French and German stock index returns. We emphasize that these illustrations are also very useful to deepen our practical knowledge as to these kinds of econometric models. For instance, as for other models, the applications of vector-half (VECH) model and DCC model were conducted in Tsuji (2017) and Tsuji (2016), respectively.

As we documented above, our general interpretations and practical case result interpretations presented in this paper shall be useful to deepen our understanding of MGARCH models and

their applications. It is important to understand some difficulties in interpreting the estimates from these kinds of models. We consider that these interpretations exhibited in this paper shall be also helpful for future empirical research using financial market data and similar econometric models. Hence, quantitative research using other data sets with clear interpretations of its result is one of our future works.

Acknowledgement

The author highly appreciates this journal for the repeated kind article invitation. In addition, the author greatly thanks Daisy Young for the kind editorial assistance to this paper. I also thank the Japan Society for the Promotion of Science Grant-in-Aid for Scientific Research for the generous financial support to this research. Further, I greatly appreciate anonymous reviewers for their highly supportive comments on this paper. Finally, I deeply thank all the Editors of this journal for their kind attention to this paper.

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