Macrothink Institute™

Business Management and Strategy
ISSN 2157-6068
2025, Vol. 16, No. 1

Optimal Commitment Strategy of the Wind-battery-hydrogen Hybrid System in the Spot Market

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Received: September 1, 2022 Accepted: November 2, 2022 Published: December 11, 2024

doi:10.5296/bms.v16i1.22368 URL: https://doi.org/10.5296/bms.v16i1.22368

Abstract

In this paper, we consider the problem of making the advance power commitments for the wind farms and the hydrogen supply plan for the hydrogen spot market, in the presence of the hybrid energy storage system with battery based and hydrogen based, mean-reverting price pocesses of power and hydrogen and the auto regressive energy generation process from winds, which extends the model in (Kim & Powell, 2011) and (Finnah & Gönsch, 2021). The problem is solved with dynamic programming algorithm with backward induction and the infinite horizon analysis. We obtained an optimal energy commitment policy and an optimal hydrogen supply plan under the above assumptions, which are indeed the extensions of the conclusion in (Kim & Powell, 2011) and (Finnah & Gönsch, 2021). Finally the property of stationary process of the hybrid storage levels corresponding to the optimal policy and plan are proved.

Keywords: Renewable energy, hydrogen storage, hybrid energy storage system, dynamic programming

1. Introduction

1.1 The Problems and Possible Solution & Improvements

A relaxed assumptions of original assumptions in (Kim & Powell, 2011) about the storage capacity and conversion losses, but ignoring the physics of energy storage. (Finnah & Gönsch,

2021) and (Garcia-Torres & Bordons, 2015) introduce two storage devices: a battery and a hydrogen based storage system for the decision making problem of the power producer. (Finnah & Gönsch, 2021) is the first to capture the decision problem of a profit maximizing power producer with multiple storage technologies in a dynamic program.

However, due to the complexity in intractability of mathematics, without giving the strict structural results and explicit formulation of the decision variables as in (Kim & Powell, 2011) and (Finnah & Gönsch, 2021) solve the problem with a backwards approximate dynamic programming algorithm with optimal computing budget allocation and obtain the numerical result finally.

In our proposal, we would go on the research under the combining assumptions of (Kim & Powell, 2011) and (Finnah & Gönsch, 2021) in electricity price process and multiple storage technologies to keep the consistence of the previous literature and try to give the analytical results, including the strict structural results and explicit formulation of the decision variables and analysis of horizon condition as the same as in (Kim & Powell, 2011). Besides, we would discuss and conclude the difference between the above main results of (Kim & Powell, 2011) and our work under the extra assumptions of multiple storage technologies.

The integration of advanced production systems and innovative decision-making frameworks has been an area of focus in recent studies, addressing challenges of efficiency, sustainability, and optimization in various industrial and technological domains. Guo et al. (2020) propose a sustainable quality control mechanism for heavy truck production processes, emphasizing plant-wide optimization techniques and quality assurance frameworks. Their study complements mathematical decision-making models like those of Kim and Powell (2011), by demonstrating real-world applications of quality control in complex production systems. Similarly, Guo et al. (2019a) introduce a quality control methodology based on the turtle diagram and evaluation model, applying these in product-service systems and aligning their work with multi-objective optimization techniques discussed in earlier decision-making studies.

The role of knowledge-driven innovation is another important area of exploration. Wang et al. (2020) introduce the "User-Knowledge-Product" Co-Creation Cyberspace Model, which emphasizes the use of cyberspace-enabled environments for fostering product innovation. This approach highlights the significance of collaborative innovation ecosystems in managing complex decision-making processes, complementing energy storage decision models like those in Kim and Powell (2011). Extending this line of research, Wang et al. (2019) examine the implications of cultural differences and user-driven product improvements in e-commerce platforms, bridging the gap between technical and behavioral optimization in product-service systems.

Data-centric methods have also gained prominence in prioritizing industrial decision-making and system improvements. Wu et al. (2018) propose a dynamic importance-performance analysis framework to identify and prioritize product-service improvements, incorporating feedback mechanisms and predictive controls to optimize decision-making under uncertainty. This framework aligns with the model predictive control techniques described by Garcia-Torres and Bordons (2015). Similarly, Guo et al. (2019b) utilize systematic G8D methods to address vibration issues in heavy trucks, which intersects with the broader effort to optimize both physical and operational parameters in production systems.

Emerging technologies like the Industrial Internet of Things (IIoT) and cloud-based systems have been explored for their transformative potential in industrial operations. Guo et al. (2020) conduct a bibliometric analysis of IIoT applications, demonstrating how these technologies advance manufacturing systems and decision-making capabilities. This research aligns with hybrid storage models discussed by Finnah and Gönsch (2021), as both emphasize the role of digital transformation and real-time data utilization in operational optimization.

In addition to technological advancements, financial decision-making and predictive modeling have emerged as essential tools for resource allocation and decision support. Zhang et al. (2018a) propose a multi-factor stock selection model utilizing machine learning techniques like LSTM and kernel support vector machines, demonstrating the effectiveness of predictive analytics in financial operations. These approaches complement the backwards approximate dynamic programming models proposed by Finnah and Gönsch (2021), integrating advanced analytical tools to enhance decision-making in complex systems.

Lastly, systematic approaches to innovation and task pricing provide insights into cost allocation and resource optimization in industrial systems. Lin et al. (2018) and Guo et al. (2019c) introduce models for task pricing in crowdsourcing platforms and product-service systems, supporting sustainable economic frameworks. These studies contribute to achieving cost-effectiveness in operational decision-making, aligning with the broader goals of optimization under multi-constraint conditions.

1.2 Assumptions

We inherit and modified the assumptions of [2, 1], by adding [4]'s modeling framework containing a battery and a hydro-gen based storage technology. It is worth mentioning that, to obtain a solution set with good properties and meaningful results, for the sake of mathematical tractability, we probably have to add necessary assumptions in our further work, whose rationality will be argued and guaranteed by literature

- Assumption 1. The power producer trades only in the continuous intraday market.
- Assumption 2. The power producer trades the hourly products right before market closure to the volume weighted average price.
- Assumption 3. The power producer is a price-taker.
- Assumption 4. The prices at the discrete intraday market following a Markov process,are assumed to be mean-reverting with stationarity in the errors in wind forecasts.

- Assumption 5. At any time, the power production during the next hour is known.forecasts.
- Assumption 6. Hydrogen can be sold instantaneous to a known price.
- Assumption 7. If energy must be stored, the share of excess energy to store in the battery would be constant.
- Assumption 8. If energy must be delivered, and the demand will be satisfied from the rest two sources available according to the following action preference rule: (1) currently produced energy, (2) energy from the battery and energy from the hydrogen storage simultaneously.
- Assumption 9. The power producer is risk-neutral.

The framework of the ideal power plant agents are modeled in figure 1.

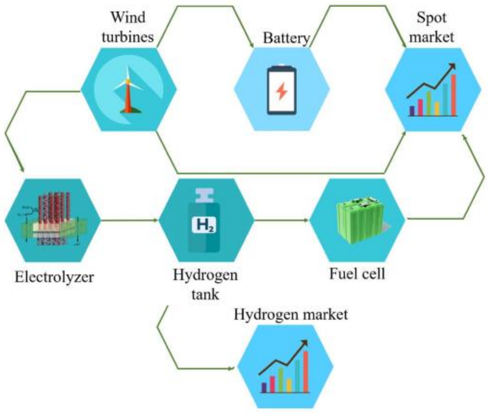

Figure 1. The framework of the ideal power plant agents

2. Model Formulation

2.1 System Parameters

- μ_p^E = mean of the spot market price of the electricity. (unit: euro/electricity unit)

- μ_p^H = mean of the spot market price of the hydrogen. (unit: euro/hydrogen unit)

- σ_p^E = standard deviation of the change in spot market price of the electricity. (unit: euro/electricity unit)

- $\sigma_p^H =$ standard deviation of the change in spot market price of the hydrogen. (unit: euro/hydrogen unit)

- $\kappa^E =$ mean-reversion parameter for the spot market price of the electricity. κ^E is proportional to the expected frequency at which the spot market price of electricity crosses the mean per unit time. (unit: $1/$ time unit)

- $\kappa^H =$ mean-reversion parameter for the spot market price of the hydrogen. κ^H is proportional to the expected frequency at which the spot market price of hydrogen crosses the mean per unit time. (unit: 1 /time unit)

- $\Delta\tau =$ time interval between decision periods.

- $m =$ slope of the penalty cost for over-commitment.

- $b =$ intercept of the penalty cost for over-commitment.(unit: euro/electricity unit)

- $\mu_Y =$ mean of the electricity generated from the wind farm per unit time. (unit: electricity unit/time unit)

- $\sigma_Y =$ standard deviation per unit time of the electricity generated from the wind farm. (unit: electricity unit/time unit)

- $\gamma =$ discount factor in the MDP model. $0 < \gamma < 1$.

2.2 Exogenous Information

The exogenous information

$$W_t = \left((Y_{t'}^{E})_{1 \le t' \le t}, P_t^{E}, P_t^{H}\right) \tag{1}$$

where

- $P_t^E =$ the spot market price for electricity during period t, which is already observable during period $t-1$, (i.e. during the time interval $[t-1, t)$). (unit: euro/electricity unit). $\forall t, P_t^E \geq 0$.

- $P_t^H =$ the spot market price for hydrogen during period t, which is already observable during period $t-1$, (i.e. during the time interval $[t-1, t)$). (unit: euro/hydrogen unit). $\forall t, P_t^H \geq 0$.

- $Y_t =$ the electricity generated from the wind turbines during the time interval $[t-1, t)$. $\forall t, Y_t \geq 0$.

For the electricity generated from the wind turbines during the time interval $[t-1, t)$:

$$Y_{t+1}^{E} = \mu_Y \Delta\tau + \sum_{i=0}^{M-1} \alpha_i (Y_{t-i}^{E} - \mu_Y \Delta\tau) + \hat{y}_{t+1} \tag{2}$$

where:

- $\hat{y}_t =$ noise that captures the random evolution of Y_{t+1}^{E}.

Specifically, we use a discrete-time version of the Ornstein-Uhlenbeck process for the spot market prices for electricity and hydrogen.

The spot market price for electricity delivered during the time interval $[t-1, t)$.

$$P_{t+1}^{E} - P_t^{E} = \kappa^{E}\left(\mu_p^{E} - P_t^{E}\right)\Delta\tau + \hat{P}_{t+1}^{E} \tag{3}$$

where:

- $\hat{P}_t^{E} =$ the noise that captures the random evolution of P_t^{E}

The spot market price for hydrogen delivered during the time interval $[t-1, t)$.

$$P_{t+1}^{H} - P_t^{H} = \kappa^{H}\left(\mu_p^{H} - P_t^{H}\right)\Delta\tau + \hat{P}_{t+1}^{H} \tag{4}$$

where:

- $\hat{P}_t^{H} =$ the noise that captures the random evolution of P_t^{H}

2.3 Decision Variables

$$x_t = (x_t^{E}, x_t^{H}) \tag{5}$$

where:

- $x_t^E =$ amount of electricity we commit to sell with delivery during time interval $[t, t+1)$, determined by signing the contract at time $t. x_t^E > 0, \forall t$.

- x_t^H : amount of hydrogen to sell with instant delivery at the end of period t. $x_t^H > 0, \forall t$.

2.4 State Variables

Let $t \in \mathbb{N}_+$be a discrete time index corresponding to the decision period. The actual time corresponding to the time index t is $t\Delta\tau$.

$$S_t = (R_t^B, R_t^H, W_t) = \left(R_t^B, R_t^H, P_t^E, P_t^H, \left(Y_{t'}^E\right)_{1\le t' \le t}\right) \qquad (6)$$

where:

- $R_t^B =$ the storage levels denoting the amount of energy stored in the battery at the beginning of period t.

- $R_t^H =$ the storage levels denoting the amount of energy stored in the hydrogen tank at the beginning of period t.

Let Ω be the set of all possible outcomes and let $\mathcal{F}$ be a σ-algebra on the set, with filtrations $\mathcal{F}_t$ generated by the information given up to time t :

$$\mathcal{F}_t = \sigma(S_0, x_0, Y_1^E, S_1, x_1, Y_2^E, S_2, x_2, \dots, Y_t^E, S_t, x_t) \tag{7}$$

$\mathbb{P}$ is the probability measure on the measure space $(\Omega, \mathcal{F})$. And We have defined the state of our system at time t as all variables that are $\mathcal{F}_t$-measurable and needed to compute our decision at time t.

2.5 State Variables

$$S_{t+1} = S_t^M(S_t, x_t, W_{t+1}) = (R_{t+1}^B(S_t, x_t), R_{t+1}^H(S_t, x_t), W_{t+1}) \tag{8}$$

with the electric power flows in and out of the storage described by the functions:

$$y_{t+1,\text{ in}}^B(S_{t+1}, x_{t+1}) = \begin{cases} \min\left\{ r(Y_{t+1}^E - x_t^E), \dfrac{R_{\max}^B - R_t^B}{\Delta\tau\rho_{in}^B} \right\}, Y_{t+1}^E \geq x_t^E \\ 0, \text{ else} \end{cases} \tag{11}$$

$$y_{t+1,\text{ out}}^B(S_{t+1}) = \begin{cases} \min\left\{ x_t^E - Y_{t+1}^E, R_t^B \dfrac{\rho_{\text{out}}^B}{\Delta\tau} \right\}, Y_{t+1}^E < x_t^E \\ 0, \text{ else} \end{cases} \tag{12}$$

$$y_{t+1,\text{ in}}^H(S_{t+1}, x_{t+1}) = \begin{cases} \min\left\{ (1-r)(Y_{t+1}^E - x_t^E), \dfrac{R_{\max}^H - R_t^H}{\Delta\tau\rho_{in}^H} \right\}, Y_{t+1}^E \geq x_t^E \\ 0, \text{ else} \end{cases} \tag{13}$$

$$y_{t+1,\text{ out}}^H(S_{t+1}) = \begin{cases} \min\left\{ x_t^E - Y_{t+1}^E - y_{t+1,\text{ out}}^B(S_{t+1}), R_t^H \dfrac{\rho_{\text{out}}^H\}}{\Delta\tau} \right\}, Y_{t+1}^E < x_t^E \\ 0, \text{ else} \end{cases} \tag{14}$$

where:

- $\forall \mathrm{i} \in \{\mathrm{B}, \mathrm{H}\}, \mathrm{j} \in \{\text{ in, out }\}, \rho_{\mathrm{j}}^{\mathrm{i}} =$ the charge/discharge efficiency of the storages.

- $\forall \mathrm{i} \in \{\mathrm{B}, \mathrm{H}\}, \mathrm{R}_{\max}^{\mathrm{i}} =$ the capacity of the storages.

For the parameters, we have:

$$m \geqslant \frac{\gamma}{\rho_{\text{out}}^{H}\ {}^{H}}, \ b \geqslant \frac{\gamma}{\rho_{\text{out}}^{H}} \mu_{p}^{E} \tag{15}$$

$$\rho_{\text{in}}^{H} \rho_{\text{out}}^{H} \leq \rho_{\text{in}}^{B} \rho_{\text{out}}^{B} \leq \frac{1-r}{r} \tag{16}$$

$$\gamma \leqslant \min \left(\frac{1-\rho_{\text{in}}^{B} \rho_{\text{out}}^{B}}{1-\rho_{\text{in}}^{B} \rho_{\text{out}}^{B} + \frac{\rho_{\text{in}}^{B} \rho_{\text{out}}^{B}}{\rho_{\text{in}}^{H} \rho_{\text{out}}^{H}}}, \frac{1-\rho_{\text{in}}^{H} \rho_{\text{out}}^{H}}{1-\rho_{\text{in}}^{H} \rho_{\text{out}}^{H} + \frac{\rho_{\text{in}}^{H} \rho_{\text{out}}^{H}}{\rho_{\text{in}}^{B} \rho_{\text{out}}^{B}}} \right) \tag{17}$$

Here, if we take $r = 1, \rho_{\text{in}}^{B} = \frac{\rho_R}{\Delta\tau}, \rho_{\text{out}}^{B} = \Delta\tau\rho_E, R_t^H = 0, \forall t,$

$$R_{t+1}^{B} = \begin{cases} R_{\max}^{B}, & \text{if } R_t^B + \rho_R(Y_{t+1}^E - x_t^E) \geqslant R_{\max}^B \\ R_t^B + \rho_R(Y_{t+1}^E - x_t^E), & \text{if } x_t^E < Y_{t+1}^E, R_t^B + \rho_R(Y_{t+1}^E - x_t^E) < R_{\max}^B \\ R_t^B - \frac{1}{\rho_E}(x_t^E - Y_{t+1}^E), & \text{if } Y_{t+1}^E \leqslant x_t^E < \rho_E R_t^B + Y_{t+1}^E \\ 0, & \text{if } x_t^E \geqslant \rho_E R_t^B + Y_{t+1}^E \end{cases} \tag{18}$$

And we also have the implicit form of the storage level:

$$R_{t+1}^{B}=\begin{cases}0, & \text{if } x_t^E - R_t^B\dfrac{\rho_{\text{out}}^{B}}{\Delta\tau} > Y_{t+1}^{E}\geq 0\\ \left(R_t^B-\dfrac{\Delta\tau}{\rho_{\text{out}}^{B}}\left(x_t^E-Y_{t+1}^{E}\right)\right), & \text{if } x_t^E > Y_{t+1}^{E}\geq x_t^E - R_t^B\dfrac{\rho_{\text{out}}^{B}}{\Delta\tau}\\ \left(R_t^B+\Delta\tau\rho_{\text{in}}^{B}\,r\left(Y_{t+1}^{E}-x_t^E\right)\right), & \text{if } x_t^E+\dfrac{R_{\max}^{B}-R_t^B}{\Delta\tau\rho_{\text{in}}^{B}\,r} > Y_{t+1}^{E}\geq x_t^E.\\ R_{\max}^{B}, & \text{if } Y_{t+1}^{E}\geq x_t^E+\dfrac{R_{\max}^{B}-R_t^B}{\Delta\tau\rho_{\text{in}}^{B}\,r}.\end{cases} \quad (19)$$

$$R_{t+1}^{H}=\begin{cases}0, & \text{if } x_t^E - R_t^B\dfrac{\rho_{\text{out}}^{B}}{\Delta\tau} - R_t^H\dfrac{\rho_{\text{out}}^{H}}{\Delta\tau} + x_t^H\dfrac{\rho_{\text{out}}^{H}}{\Delta\tau} > Y_{t+1}^{E}\geq 0\\ \left(R_t^H-\dfrac{\Delta\tau}{\rho_{\text{out}}^{H}}\left(x_t^E-Y_{t+1}^{E}-R_t^B\dfrac{\rho_{\text{out}}^{B}}{\Delta t}\right)-x_t^H\right) & \text{if } x_t^E > Y_{t+1}^{E} \geq x_t^E - R_t^B\dfrac{\rho_{\text{out}}^{B}}{\Delta\tau} - R_t^H\dfrac{\rho_{\text{out}}^{H}}{\Delta\tau} + x_t^H\dfrac{\rho_{\text{out}}^{H}}{\Delta\tau}\\ \left(R_t^H+\Delta\tau\rho_{\text{in}}^{H}(1-r)(Y_{t+1}^{E}-x_t^E)-x_t^H\right) & \text{if } x_t^E+\dfrac{R_{\max}^{H}-R_t^H}{\Delta\tau\rho_{\text{in}}^{H}(1-r)} > Y_{t+1}^{E}\geq x_t^E\\ \left(R_{\max}^{H}-x_t^H\right), & \text{if } Y_{t+1}^{E}\geq x_t^E+\dfrac{R_{\max}^{H}-R_t^H}{\Delta\tau\rho_{\text{in}}^{H}(1-r)}.\end{cases} \quad (20)$$

2.6 Decision Variables

The profit we make during the time interval $[t, t+1)$ is given by

$$\hat{C}_{t+1} = P_{t+1}^{H}x_t^H + \Delta\tau P_{t+1}^{E}x_t^E \quad (21)$$
$$-\Delta\tau Q_{t+1}^{E,u}\max\{x_t^E - Y_{t+1}^{E} - y_{t+1,\text{out}}^{B} - y_{t+1,\text{out}}^{H}, 0\} \quad (21)$$

where:

- $Q_t^{E,o} =$ the penalty one has to pay in the case of electricity shortage in the current period. This affine penalty $\left(Q_{t+1}^{E,u} = mP_{t+1}^{E} + b\right)$ is sufficient to ensure the concavity of the stochastic optimization problem.

Define the contribution, or the reward function:

$$C(S_t, x_t) := \mathbb{E}\left[\hat{C}_{t+1} \mid S_t, x_t\right]$$
$$= x_t^H\left[\mu_p^H + (1 - \kappa^H \Delta\tau)\left(P_t^H - \mu_p^H\right)\right]$$
$$+\Delta\tau\left[\mu_p^E + (1 - \kappa^E \Delta\tau)\left(P_t^E - \mu_p^E\right)\right] \times$$
$$-\Delta\tau b \int_{0 \leq y < x_t^E - R_t^B \frac{\rho_{\text{out}}^B}{\Delta\tau} - R_t^H \frac{\rho_{\text{out}}^H}{\Delta\tau}} F_t(y) dy$$

where:

- $F_t(y) = \mathbb{P}[Y_{t+1} \leqslant y \mid \mathcal{F}_t]$

See the proof in Appendix 6.1 for the derivation of (22).

2.7 Objective Function

Let Π be the set of all policies. A policy is an $\mathcal{F}_t$-measurable function $X^{\pi}(S_t)$ that describes the mapping from the state at time t, S_t, to the decision at time t, x_t. For each $\pi \in \Pi$, let

$$G_t^{\pi}(S_t) := \mathbb{E}\left[\sum_{t'=t}^{T} \gamma^{t'-t} C\left(S_{t'}, X^{\pi}(S_{t'})\right) \mid S_t\right], \forall 0 \leqslant t \leqslant T \quad (23)$$

where:

- $\gamma \in (0,1)$ is the discount factor and T indicates the end of the horizon.

The objective, then, is to find an optimal policy $\pi = \pi^*$ that satisfies:

$$G_t^{\pi^*}(S_t) = \sup_{\pi \in \Pi} G_t^{\pi}(S_t), \forall 0 \leqslant t \leqslant T \tag{24}$$

In conclusion we give the temporal resolution of the problem formulation, as shown in figure 2.

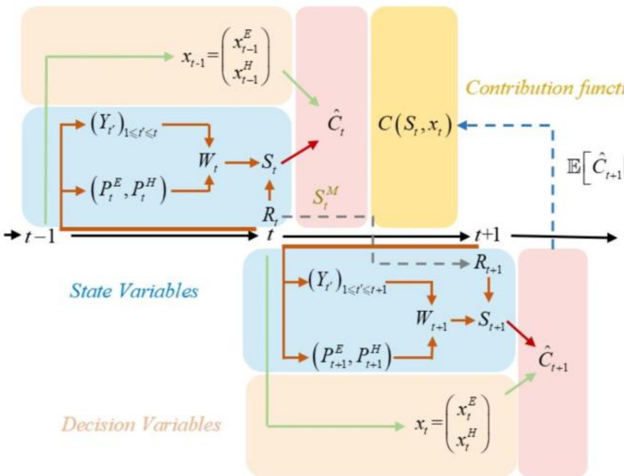

Figure 2. The events description

3. Model Analysis

In this section, we will adopt the same methodology in (Kim & Powell, 2011). to solve the dynamic optimization problem, under extended assumptions of multiple storage technologies proposed by (Finnah & Gönsch, 2021).

Specially, to obtain the closed-form representation of the optimal policy, we need assumptions on the probability distribution of the spot market price for electricity and hydrogen and wind energy, limit on the storage size, and the decision period intervals.

3.1 Primary Analysis

3.1.1 The Stochastic Process of Price for Electricity and Hydrogen and Wind Energy

First, we assume that $(\hat{P}_t^E)_{t\geqslant 0}, (\hat{P}_t^H)_{t\geqslant 0}$ and $(\hat{y}_t)_{t\geqslant 1}$ are independent in $(\Omega, \mathcal{F}, \mathbb{P})$. It is well known that the prices of the electricity and hydrogen mainly depend on the demand as well as the main source of energy that is controllable.

Following the assumption 4, as the same as the one in [1], we assume $(\hat{P}_t^E)_{t\geqslant 0}$, $(\hat{P}_t^H)_{t\geqslant 0}$ are both i.i.d. with distribution $\mathcal{N}(0, \sigma_P^{E2})$ and $\mathcal{N}(0, \sigma_P^{H2})$ respectively. Then, $(\hat{P}_t^E)_{t\geqslant 0}, (\hat{P}_t^H)_{t\geqslant 0}$ are both standard mean-reverting process and

$$\begin{aligned} \mathbb{E}[P_{t+n}^E \mid \mathcal{F}_t] &= \mu_p^E + (1 - \kappa_E \Delta\tau)^n (P_t^E - \mu_p^E), \quad \forall n, t \in \mathbb{N}_+ \\ \mathbb{E}[P_{t+n}^H \mid \mathcal{F}_t] &= \mu_p^H + (1 - \kappa_H \Delta\tau)^n (P_t^H - \mu_p^H), \quad \forall n, t \in \mathbb{N}_+ \end{aligned} \tag{26}$$

Where

- $\frac{P_{t,t+1}^H}{P_{t,t+1}^t} > \frac{\gamma}{\rho_{in}^H}$ are considered.

See the proof in Appendix 6.2 for the derivation of (25) and (26).

We apply the optimal policy derived under the assumption of uniformly distributed $(\hat{y}_t)_{t\geqslant 1}$ to the data generated from truncated Gaussian distributions. Then, given $\mathcal{F}_t, Y_{t+1}^E \sim \mathcal{U}(\theta_t, \theta_t + \beta)$, where:

- $\beta := 2\sqrt{3}\sigma_Y \Delta\tau$

- $\theta_t := \mu_Y \Delta\tau + \sum_{i=0}^{M-1} \alpha_i (Y_{t-i} - \mu_Y \Delta\tau) - \frac{\beta}{2}$, $\forall t$

The cumulative density function (CDF) of Y_{t+1} computed at time t is given by

$$F_t(y) = \mathbb{P}[Y_{t+1}^E \leqslant y \mid \mathcal{F}_t] = \begin{cases} 0, & \text{if } y < \theta_t \\ \dfrac{y - \theta_t}{\beta} & \text{if } \theta_t \leqslant y \leqslant \theta_t + \beta \\ 1, & \text{if } y > \theta_t + \beta \end{cases} \tag{27}$$

3.1.2 The Size of the Storage of the Battery and the Hydrogen Based Storage System

Here we add the assumption of the assumption on the size of the storage of the battery and the hydrogen based storage system. We propose that the size of the storage of the Battery and the Hydrogen based storage system be determined in comparison to $\nu^E, (\nu^H$, respectlively), given by:

$$\nu^j := \min\left\{\overline{R_{\max,1}^j}, \overline{R_{\max,2}^j}\right\}, \forall j \in \{B(\text{ or } E), H\} \tag{28}$$

where,

$$\overline{R_{\max,1}^B} := \frac{\Delta\tau \rho_{\text{in}}^B \beta(m-1)}{m - \rho_{\text{in}}^B \rho_{\text{out}}^B r\gamma(1 - \kappa^E \Delta\tau)} \tag{29}$$

$$\overline{R_{\max,2}^B} := \frac{\Delta\tau \rho_{\text{in}}^B r\beta b - \overline{R_{\max,1}^B} \rho_{\text{in}}^B \rho_{\text{out}}^H r}{m - \rho_{\text{in}}^B \rho_{\text{out}}^B r\gamma(1 - \kappa^E \Delta\tau)} \tag{30}$$

$$\overline{R_{\max,1}^H} := \frac{\Delta\tau \rho_{in}^H (1-r)\beta(m-1)}{m - \rho_{\text{in}}^H \rho_{\text{out}}^H (1-r)\gamma(1 - \kappa^E \Delta\tau)} \tag{31}$$

$$\overline{R_{\max,2}^B} := \frac{\Delta\tau \rho_{\text{in}}^H (1-r)\beta b - \overline{R_{\max,1}^H} \rho_{\text{in}}^H \rho_{\text{out}}^B r}{m - \rho_{\text{in}}^B \rho_{\text{out}}^B (1-r)\gamma(1 - \kappa^E \Delta\tau)} \tag{32}$$

It is obvious that as the penalty factors m and b become larger, we need to allow for a larger storage because our commitment level will be more conservative and we will end up storing more energy. Also, if the round-trip efficiency of the storage $\rho_{\text{out}}^j \rho_{\text{in}}^j$ is small, we must allow for a larger storage in order to compensate for the energy that will be lost in conversion. Next, because $\gamma\mu_p^j$ is the discounted expected spot market price of the electricity, if $\gamma\mu_p^j$ is small, we need to allow for a larger storage because our commitment level will be more conservative.

For this paper, we assume

$$R_{\max}^j \leqslant \min\left\{\overline{R_{\max,1}^j}, \overline{R_{\max,2}^j}\right\}, \forall j \in \{B(\text{ or } E), H\} \tag{33}$$

3.1.3 Decision Period Interval

In this section we would compute the lower bound of the decision period interval.

We can rearrange the terms from (33):

$$\max\left[\begin{array}{c}\dfrac{R_{\max}^j\left(m-\rho_{out}^j\rho_{in}^j\gamma\right)}{2\sqrt{3}(m-1)\rho_{in}^j\sigma_Y - R_{\max}\rho_{out}^j\rho_{in}^j\gamma\kappa^j}, \\ \dfrac{R_{\max}^j}{2\sqrt{3}\rho_{in}^j\sigma_Y b - R_{\max}^j\rho_{out}^j\rho_{in}^j\gamma\kappa^j\mu_p^j}\end{array}\right] \leqslant \frac{1}{\kappa^j}, \forall j \in \{B(\text{ or } E), H\} \tag{34}$$

We assume that the time interval $\Delta\tau$ between our decision periods satisfies the following:

$$\max\left[\frac{R_{\max}^j\left(m-\rho_{\text{out}}^j\rho_{\text{in}}^j\gamma\right)}{2\sqrt{3}(m-1)\rho_{\text{in}}^j\sigma_Y - R_{\max}^j\rho_{\text{out}}^j\rho_{\text{in}}^j\gamma\kappa^j}\right. \\ \left.\frac{R_{\max}^j b}{2\sqrt{3}\rho_{\text{in}}^j\sigma_Y b - R_{\max}^j\rho_{\text{out}}^j\rho_{\text{in}}^j\gamma\kappa^j\mu_p^j}\right] \leqslant \Delta\tau \leqslant \frac{1}{\kappa^j}, \forall j \in \{B(\text{ or } E), H\} \tag{35}$$

Equivalently

$$\min_{j\in\{B(\text{ or }E),H\}} \max\left[\frac{R_{\max}^{j}(m-\rho_{\text{out}}^{j}\rho_{\text{in}}^{j}\gamma)}{2\sqrt{3}(m-1)\rho_{\text{in}}^{j}\sigma_{Y}-R_{\max}^{j}\rho_{\text{out}}^{j}\rho_{\text{in}}^{j}\gamma\kappa^{j}} \frac{R_{\max}^{j}b}{2\sqrt{3}\rho_{\text{in}}^{j}\sigma_{Y}b-R_{\max}^{j}\rho_{\text{out}}^{j}\rho_{\text{in}}^{j}\gamma\kappa^{j}\mu_{p}^{j}}\right] \leqslant \Delta\tau \leqslant \min_{j\in\{B(\text{ or }E),H\}}\frac{1}{\kappa^{j}} \tag{36}$$

The lower bound can be rearranged to be written as: $\forall j \in \{B(\text{ or } E), H\}$,

$$R_{\max}^{j} \leqslant \rho_{\text{in}}^{j}\beta\min\left[\frac{m-1}{m-\rho_{\text{out}}^{j}\rho_{\text{in}}^{j}\gamma(1-\kappa^{j}\Delta\tau)},\frac{b}{b+\rho_{\text{out}}^{j}\rho_{\text{in}}^{j}\gamma\kappa^{j}\Delta\tau\mu_{p}^{j}}\right] \tag{37}$$

See the proof in Appendix 6.3 for the derivation from (33) to (37).

Because we have a limit on the size of our storage, if $\Delta\tau$ is too large, the amount of electricity that is produced between our decisions can be too large and we are likely to lose energy due to the storage being full. (36) gives us a reasonable decision period time interval $\Delta\tau$.

3.1.4 Structural Results

According the above important formulations, we can explore the important structural results of the value function. Let $V_t^{\pi}(S_t)$ be a function that satisfies

$$V_T^{\pi}(S_T) = C\big(S_T, X_T^{\pi}(S_T)\big) \tag{38}$$

$$V_t^{\pi}(S_t) = C\big(S_t, X_t^{\pi}(S_t)\big) + \gamma\mathbb{E}[V_{t+1}^{\pi}(S_{t+1}) \mid S_t], \forall 0 \leqslant t \leqslant T-1 \tag{39}$$

Then, $V_t^{\pi}(S_t) = G_t^{\pi}(S_t), \forall 0 \leqslant t \leqslant T$. For $0 \leqslant t \leqslant T$, let $V_t(S_t)$ satisfy the following:

$$V_t^x(S_t, x) := \mathbb{E}[V_{t+1}(S_{t+1}) \mid S_t, x], \ \forall 0 \leqslant t \leqslant T \quad (40)$$

Let

$$x_t^* : = \arg \max_{x_t \in \mathbb{R}_+ \times \mathbb{R}_+} \{C(S_t, x) + \gamma V_t^x(S_t, x)\} \quad (41)$$

$$= X^{\pi^*}(S_t), \ \forall 0 \leqslant t \leqslant T \quad (41)$$

Then,

$$V_t(S_t) = \max_{x_t \in \mathbb{R}_+ \times \mathbb{R}_+} \{C(S_t, x) + \gamma V_t^x(S_t, x)\} \quad (42)$$

$$= C(S_t, x_t^*) + \gamma V_t^x(S_t, x_t^*), \ \forall 0 \leqslant t \leqslant T \quad (42)$$

We can explore some properties at the end of the horizon.
Property 1. The first order derivative of value function at the end of the horizon.

$$\frac{d}{dR_T^B} V_T(S_T) = \frac{d}{dR_T^B} C(S_T, x_T^*) \quad (43)$$

$$= \rho_{\text{out}}^B \left[\mu_p^E + (1 - \kappa^E \Delta\tau)(P_T^E - \mu_p^E)\right] \quad (43)$$

$$\frac{d}{dR_T^H} V_T(S_T) = \frac{d}{dR_T^H} C(S_T, x_T^*) \quad (44)$$

$$= \rho_{\text{out}}^H \left[\mu_p^E + (1 - \kappa^E \Delta\tau)(P_T^E - \mu_p^E)\right] \quad (44)$$

Property 2. The second order derivative of value function at the end of the horizon.

$$\frac{d^2}{dR_T^{B2}} V_T(S_T) = 0 \quad (45)$$

$$\frac{d^2}{dR_T^{H2}} V_T(S_T) = 0 \quad (46)$$

$$\frac{d^2}{dR_T^B dR_T^H} V_T(S_T) = \frac{d^2}{dR_T^H dR_T^B} V_T(S_T) = 0 \quad (47)$$

See the proof in Appendix 6.4 for the deduction of (43) to (46).

Then, $\forall 0 \leqslant t \leqslant T-1$, we have:

Structural Result 1. $C(S_t, x) + \gamma V_t^x(S_t, x)$ is a concave function of on $(x_t^E, x_t^H, R_t^E, R_t^H)$.

Structural Result 2. The optimal decision x_t^* is feasible and finite and

$$D_{(x_t^E, x_t^H)}[C(S_t, x_t^*) + \gamma V_t^x(S_t, x_t^*)] = \mathbf{0}. \tag{48}$$

Structural Result 3.

$$\begin{aligned}\frac{d}{dR_t^B} V_t(S_t) = \rho_{\text{out}}^B \left[\mu_p^E + (1-\kappa^E \Delta\tau)(p_t^E - \mu_p^E)\right] \\ + \left(\frac{\rho_{\text{out}}^B}{\Delta\tau}\frac{\partial R_{t+1}^H}{\partial x_t^E} + \frac{\partial R_{t+1}^H}{\partial R_t^B}\right)\frac{d}{dR_{t+1}^H} V_{t+1}(S_{t+1}) \Bigg| S_t, x_t^*\Bigg].\end{aligned}$$

$$\begin{aligned}\frac{d}{dR_t^H} V_t(S_t) = \rho_{\text{out}}^H \left[\mu_p^E + (1-\kappa^E \Delta\tau)(p_t^E - \mu_p^E)\right] \\ + \left(\frac{\rho_{\text{out}}^H}{\Delta\tau}\frac{\partial R_{t+1}^H}{\partial x_t^E} + \frac{\partial R_{t+1}^H}{\partial R_t^H}\right)\frac{d}{dR_{t+1}^H} V_{t+1}(S_{t+1}) \Bigg| S_t, x_t^*\Bigg]\end{aligned}$$

Structural Result 4. $V_t(S_t)$ is a concave function of (R_t^E, R_t^H).

Structural Result 5.

$$\rho_{\text{out}}^B \left[\mu_p^E + (1-\kappa^E \Delta\tau)(p_t^E - \mu_p^E)\right] \leqslant \frac{d}{dR_t^B} V_t(S_t) \leqslant \frac{1}{\rho_{\text{in}}^B}\left[\mu_p^E + (1-\kappa^E \Delta\tau)(p_t^E - \mu_p^E)\right]$$

$$\rho_{\text{out}}^H \left[\mu_p^E + (1-\kappa^E \Delta\tau)(p_t^E - \mu_p^E)\right] \leqslant \frac{d}{dR_t^H} V_t(S_t) \leqslant \frac{1}{\rho_{\text{in}}^H}\left[\mu_p^E + (1-\kappa^E \Delta\tau)(p_t^E - \mu_p^E)\right]$$

See the idea and outline of the proof in Appendix 6.5 for the structural result 1 to 5 .

Business Management and Strategy
ISSN 2157-6068
2025, Vol. 16, No. 1

3.2 Main Result-Infinite-Horizon Analysis

3.2.1 Statistics and Data Analysis

In this part, we should get the optimal policy, when the electricity generated from the wind farm. In this section, we derive the marginal value function and the corresponding optimal policy for advance energy commitment that maximizes the expected revenue in the infinite-horizon case. However, although we can obtain the value of always having a storage as shown in this paper, it is important to note that the cost of always having storage is not the cost of installing the storage once in the beginning. Batteries have finite lifetime, and we might have to reinstall them every 10 years, for example. We let $T \longrightarrow \infty$ and drop the index t from the value function:

$$V(S_t) = \lim_{T\to\infty} \mathbb{E}\left[\sum_{t'=t}^{T} \gamma^{t'-t} C\left(S_{t'}, X^{\pi^*}(S_{t'})\right) \mid S_t\right] \tag{53}$$

Then, $V(S_t)$ satisfies

$$V_t(S_t) = \max_{x_t \in \mathbb{R}_+ \times \mathbb{R}_+} \{C(S_t, x) + \gamma V_t^x(S_t, x)\} \tag{54}$$

$$= C(S_t, x_t^*) + \gamma V_t^x(S_t, x_t^*), \ \forall 0 \leqslant t \leqslant T \tag{54}$$

Therefore, in order to compute x_t^*, we only need to know the derivative of $V(S_{t+1})$ with respect to R_{t+1}, and we do not need to know $V(S_{t+1})$ itself. To derive $(d/dR_{t+1})V(S_{t+1})$, we need the following lemma:

Lemma 1.

$$x_t^{E*} + \frac{R_{\max}^B - R_t^B}{\Delta\tau\rho_{in}^B r} \leqslant \theta_t + \beta,\ \forall t \tag{55}$$

$$x_t^{E*} + \frac{R_{\max}^H - R_t^H}{\Delta\tau\rho_{in}^H (1-r)} \leqslant \theta_t + \beta,\ \forall t \tag{56}$$

See the idea and outline of the proof in Appendix 232 for the optimal policy (232). Then from the main assumptions and structural results, we can have:

Theorem 1.The optimal policy, when the electricity generated from the wind farm is uniformly distributed from θ_t to $\theta_t + \beta$, is given by

x_t^{E*}

$$x_t^{H*} = R_{\max}^H - \frac{\beta\rho_{in}^H(1-r)}{\gamma(\theta_1+\theta_2)\kappa^E}\ln(1-\kappa^E\Delta\tau) \tag{58}$$

where,

$$\theta_1 = 1 - \rho_{\text{out}}^{B} \rho_{\text{in}}^{B} r \tag{59}$$

$$\theta_2 = 1 - \rho_{\text{in}}^{H} \rho_{\text{out}}^{H} (1-r) \tag{60}$$

$$\text{Min} = \min\left(\frac{R_{\max}^{B} - R_t^{B}}{\Delta\tau\rho_{in}^{B} r}, \frac{R_{\max}^{H} - R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}\right) \tag{61}$$

$$B = R_t^{B} \frac{\rho_{\text{out}}^{B}}{\Delta\tau\rho_{\text{out}}^{H} \rho_{\text{in}}^{H} (1-r)} \tag{62}$$

$$H = R_t^{H} \frac{\rho_{\text{out}}^{H}}{\Delta\tau\rho_{\text{out}}^{B} \rho_{\text{in}}^{B} r} \tag{63}$$

$$K_1 = 1 + \frac{\rho_{\text{out}}^{B} \rho_{\text{in}}^{B} r}{(\theta_1 + \theta_2)^2}\left[\theta_1 \left\{1 - \exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{B}}{\Delta\tau\rho_{\text{in}}^{B} r}\right]\right.\right. \tag{64}$$

$$+\exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{B} - R_t^{B}}{\Delta\tau\rho_{in}^{B} r}\right] 1_{0<r<1} \tag{64}$$

$$\left. -\exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{B}}{\Delta\tau\rho_{in}^{B} r}\right] 1_{0<r<1}\right\} \tag{64}$$

$$+\theta_2 \left\{\exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{H} - R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)} v - \frac{R_{\max}^{B} - R_t^{B}}{\Delta\tau\rho_{in}^{B} r}\right]\right. \tag{64}$$

$$\left.\left. -\exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{H} - R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}\right] 1_{0<r<1}\right\}\right] \tag{64}$$

$$+\frac{\rho_{\text{out}}^{H} \rho_{\text{in}}^{H} (1-r)}{(\theta_1 + \theta_2)^2}\left[\theta_1 \left\{\exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{B} - R_t^{B}}{\Delta\tau\rho_{\text{in}}^{B} r} - \text{Min}\right]\right.\right. \tag{64}$$

$$\left. -\exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{B} - R_t^{B}}{\Delta\tau\rho_{in}^{B} r}\right]\right\} \tag{64}$$

$$+\theta_2 \left\{1 - \exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{H} - R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)} + H\right]\right. \tag{64}$$

$$+2\exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{H} - R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)} - B\right] \tag{64}$$

$$\left.\left. -2\exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{H} - R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}\right]\right\}\right] \tag{64}$$

$$-\frac{\gamma\rho_{\text{out}}^{H}\theta_1}{\Delta\beta(\theta_1 + \theta_2)} R_t^{H} \exp\left[\frac{\gamma}{\beta}(\theta_1 + \theta_2)\frac{R_{\max}^{B}}{\Delta\tau\rho_{in}^{B} r}\right] \tag{64}$$

$$K_2 = 1 + \frac{\rho_{\text{out}}^{B} \rho_{\text{in}}^{B} r}{(\theta_1 + \theta_2)^2}\left[\theta_1 \left\{1 - \exp\left[\frac{\gamma}{\beta}(1 - \kappa^E \Delta\tau)(\theta_1 + \theta_2)\frac{R_{\max}^{B}}{\Delta\tau\rho_{in}^{B} r}\right]\right.\right. \tag{65}$$

$$-\exp\left[\frac{\gamma}{\beta}(1 - \kappa^E \Delta\tau)(\theta_1 + \theta_2)\frac{R_{\max}^{B} - R_t^{B}}{\Delta\tau\rho_{in}^{B} r}\right] 1_{0<r<1} \tag{65}$$

$$
\begin{aligned}
&+\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{B}}{\Delta\tau\rho_{in}^{B}r}\right]1_{0<r<1}\bigg\}\\
&+\theta_2\left\{\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{H}-R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}-\frac{R_{\max}^{B}-R_t^{B}}{\Delta\tau\rho_{in}^{B}r}\right]\right.\\
&\left.-\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{H}-R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}\right]1_{0<r<1}\right\}\bigg]\\
&+\frac{\rho_{\text{out}}^{H}\rho_{\text{in}}^{H}(1-r)}{(\theta_1+\theta_2)^2}\left[\theta_1\left\{\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{B}-R_t^{B}}{\Delta\tau\rho_{\text{in}}^{B}r}-\text{Min}\right]\right.\right.\\
&\left.-\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{B}-R_t^{B}}{\Delta\tau\rho_{in}^{B}r}\right]\right\}\\
&+\theta_2\left\{1-\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{H}-R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}+H\right]\right.\\
&+2\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{H}-R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}-B\right]\\
&\left.\left.-2\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{H}-R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}\right]\right\}\right]\\
&-\frac{\gamma(1-\kappa^{E}\Delta\tau)\rho_{\text{out}}^{H}\theta_1}{\Delta\beta(\theta_1+\theta_2)}R_t^{H}\exp\left[\frac{\gamma}{\beta}(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{R_{\max}^{B}}{\Delta\tau\rho_{in}^{B}r}\right]
\end{aligned}
$$

Theorem 2.

$$
\begin{aligned}
\frac{d}{dR_t^{B}}V(S_t)=&\rho_{\text{out}}^{B}\mu_p^{E}\left\{\frac{\theta_1}{\theta_1+\theta_2}\exp\left[\gamma(\theta_1+\theta_2)\frac{1}{\beta}\left(\frac{R_{\max}^{B}-R_t^{B}}{\Delta\tau\rho_{\text{in}}^{B}r}\right)\right]\right.\\
&\left.+\frac{\theta_2}{\theta_1+\theta_2}\exp\left[\gamma(\theta_1+\theta_2)\frac{1}{\beta}\left(\frac{R_{\max}^{H}-R_t^{H}}{\Delta\tau\rho_{\text{in}}^{H}(1-r)}\right)\right]\right\}\\
&+\rho_{\text{out}}^{B}(p_t^{E}-\mu_p^{E})(1-\kappa^{E}\Delta\tau)\\
&\cdot\left\{\frac{\theta_1}{\theta_1+\theta_2}\exp\left[\gamma(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{1}{\beta}\left(\frac{R_{\max}^{B}-R_t^{B}}{\Delta\tau\rho_{\text{in}}^{B}r}\right)\right]\right.\\
\frac{d}{dR_t^{H}}V(S_t)=&\rho_{\text{out}}^{H}\mu_p^{E}\left\{\frac{\theta_1}{\theta_1+\theta_2}\exp\left[\gamma(\theta_1+\theta_2)\frac{1}{\beta}\left(\frac{R_{\max}^{B}-R_t^{B}}{\Delta\tau\rho_{\text{in}}^{B}r}\right)\right]\right.\\
&\left.+\frac{\theta_2}{\theta_1+\theta_2}\exp\left[\gamma(\theta_1+\theta_2)\frac{1}{\beta}\left(\frac{R_{\max}^{H}-R_t^{H}}{\Delta\tau\rho_{in}^{H}(1-r)}\right)\right]\right\}\\
&+\rho_{\text{out}}^{H}(p_t^{E}-\mu_p^{E})(1-\kappa^{E}\Delta\tau)\\
&\cdot\left\{\frac{\theta_1}{\theta_1+\theta_2}\exp\left[\gamma(1-\kappa^{E}\Delta\tau)(\theta_1+\theta_2)\frac{1}{\beta}\left(\frac{R_{\max}^{B}-R_t^{B}}{\Delta\tau\rho_{\text{in}}^{B}r}\right)\right]\right.
\end{aligned}
$$

3.2.2 Statistics and Data Analysis

In order to obtain a closed-form expression for the expected value of storage, we must analyze the dynamics of our system at the steady state and prove the the stationary distribution of the storage level.

Proposition 1. At steady state,

$$P_t^j \sim \mathcal{N}\left(\mu_p^j, \frac{\sigma_p^{j2}}{1-(1-\kappa^j \Delta\tau^j)^2}\right), \forall j \in \{B(\text{ or } E), H\} \tag{68}$$

Proposition 2. The distribution of R_t corresponding to the steady state is stationary.

4. Conclusion

In our work, we provide the analytical results of the decision problem of a profit maximizing wind power producer with a hybrid energy storage system with battery based and hydrogen based, including: the optimal advance power commitments for the wind farms and the optimal hydrogen supply plan for the hydrogen spot market, in the presence of the hybrid energy storage system with battery based and hydrogen based, mean-reverting price processes of power and hydrogen and the auto regressive energy generation process from winds, which extends the model and results in (Kim & Powell, 2011) and (Finnah & Gönsch, 2021). Finally the property of stationary process of the hybrid storage levels corresponding to the optimal policy and plan are proved.

A number of assumptions such as stationarity in the wind and price processes, and the assumption of uniformly distributed errors in the wind forecast guarantee the ideal properties or our model. It would be nice if we could show that the optimal policy always has a form similar to the newsvendor problem as shown in (Kim & Powell, 2011), regardless of the distribution of wind energy. Another dimension arises in risk mitigation when modeling heavy-tailed behaviors in electricity prices and hedrygen prices. Moreover, there will always be a need for accurate models that will have to be solved using numerical methods, but at the same time we feel that there will also be interest in analytical models that are easy to compute and that provide insights into trade-offs between parameters.

Acknowledgments

Not applicable.

Authors contributions

Not applicable.

Funding

Not applicable.

Competing interests

Not applicable.

Informed consent

Obtained.

Ethics approval

The Publication Ethics Committee of the Macrothink Institute.

The journal's policies adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

Provenance and peer review

Not commissioned; externally double-blind peer reviewed.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement

No additional data are available.

Open access

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