Equity Capital-Structure-Based Evaluation Method

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Abstract

The main purpose of this paper is to theoretically compare three structural models presenting several similarities and using financial statements within the context of real options theory. The models are those suggested by i) Leland (1994); ii) Goldstein, Ju and Leland (2001) and iii) Sarkar and Zapatero (2003). The analysis emphasizes convergence conditions of the three models based on their respective dynamic equations. The results show that the first two models represent special cases of the third one. The paper also presents a new equity and debt valuation method.

Keywords: CSR Structural model, F230inancial statement, Equity, EBIT, Mean reversion, Contingent claim, Convergence

1. Introduction

Very few capital-structure-based models can be used to evaluate companies. Also, most of them assume no correlation between the market and the companies' financial statements, which we believe explains the market inefficiency. Our paper presents a new structural evaluation model based on mean reversion assumptions and using financial statements information. That being said, we start by comparing three existing structural models, namely i) Leland (1994), ii) Goldstein, Ju and Leland (2001) and iii) Sarkar and Zapatero (2003). Although these models present some discrepancies, they also have some similarities and this is what makes the investigation of their convergence conditions very interesting. While Leland (1994) and Goldstein, Ju and Leland (2001) use log normal processes for their respective state variables, Sarkar and Zapatero(2003) suggest a mean reversion process. In our paper, we emphasize the conditions that make their contingent claims expressions similar.

The rest of the paper is organized as follows: Section II presents a literature review. Section III focuses on our methodology. Section IV discusses the convergence between the three models analyzed. Section V emphasizes our new evaluation method. Section VI presents our conclusions. Proofs will be supplied upon request.

2. Literature Review

Very few capital-structure-based models can be used to assess securities. Some emphasize debt evaluation and others the contingent claims approach by using the arbitrage between tax shields and bankruptcy costs. Merton (1974) was actually the first author to consider equity as an option. The underlying asset of this option is the firm's value while debt represents its strike price. Using a dynamic lognormal process equation, the author obtains a closed form solution for his partial differential equation (PDE), similar to the Black-Scholes (1973) equation.

Black and Cox (1976) reexamineMerton's model but address subordinated debt and the company's solvency relationship issue, in order to better control a possible future financial distress with regard to jump processes and bankruptcy threshold.

In the same vein, Zhou (2001) suggests a hybrid model that includes both continuous and jump diffusion processes, in order to instantaneously capture new information that could allow for an instantaneous bonds and equity evaluation with regard to bankruptcy threshold.

Long staffand Schwartz (1995) also use a process similar to Merton's (1974), with the same state variable; however, they propose a term structure of interest following a mean reversion process like Vasicek(1977) and a non-zero-correlation between interest rate and assets value. Their closed form solution is applicable for both fixed and variable rates of debt interest.

Brennan and Schwartz's (1978) PDE closed form solution is also similar to Merton's one but can only be solved numerically because of its finite life debt. The two authors argue that debt should be renewable and that its net effect on the company value is a consequence of the arbitrage between tax shields and bankruptcy costs.

Brennan and Schwartz (1984) add that for solvency, debt should have a net positive effect on the company value but a negative one in the case of distress, due to agency costs that eliminate tax shields benefits should this last situation occur. Similarly, Ju et al. (2005) use assets value as a state variable that follows a lognormal process and show that the company can target a capital structure level that allows it to choose the best time to refinance the debt.

Leland and Toft (1996) extend the debate by considering that under a finite debt assumption, time becomes important when determining the optimal capital structure and the arbitrage between tax shields, bankruptcy costs and agency costs.

According to Sarkar and Mauer (2005), these agency costs emanate from conflicts between equity holders and creditors about the best time to invest. Indeed, equity holders always choose the timing that maximizes their wealth and by doing so, they transfer the risk to creditors, which contributes to raising agency costs and diminishing company value.

The pioneer of the second structural models generation is Leland (1994) who proposes a new approach based on the arbitrage between tax shields and bankruptcy costs. He argues that these last two variables should be considered as contingent claims for his model's PDE. Consequently, a relationship between optimal capital structure and market value could be established. Thus, equity holders continue holding company shares while reimbursing debt and by doing so, they actually hold a call option whose underlying asset and strike price are, respectively, the company value and the perpetual debt that pays a continuous coupon as long as the company remains solvent. Indeed, in the case of bankruptcy, debt holders take possession of company assets.

Goldstein, Ju and Leland (2001) find that Leland's (1994) model is static and presents some uncertainty induced by the non-negotiability of the assets value. According to the three authors, the latter assumes a positive relationship between taxes and tax shields, presumed to increase equity value. That implies a positive relationship between taxes and equity, which is aberrant. Hence, a static capital structure overestimates the instantaneous growth of the company value, which implies an unrealistic low threshold of bankruptcy probability. The three authorssuggest that taxes be considered as contingent claims to be handled like equity and debt. Hence, their value becomes proportional to the company's. Considering a lognormal process and EBIT as state variable, they derive a dynamic equation and obtain a closed form solution for their PDE.

Similarly, Sarkar and Zapatero (2003) use an EBIT state variable that, however, follows a mean reversion process, which helps to avoid the ambiguity surrounding the relationship between taxes and the company value. They also point out the irrelevance of Leland's (1994) proportionality assumption between the continuous coupon and the company value, which implies a non-constant value of the former. The two authors assume a constant perpetual coupon that is insensitive to company earning variations but not to the company's capital structure.

Hackbarth, Henessy and Leland (2007) extend Leland (1994) and Goldstein, Ju and Leland (2001) by considering two debt categories: bank debt and market debt. According to them, the flexibility offered by banking debt reduces bankruptcy costs and then helps to optimize the arbitrage relationship between those costs and the tax shield. Thus, small firms should use bank debt to finance their projects while large firms that are looking for an optimal leverage, should use a combination of bank debt and market debt while prioritizing the first one[1].

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[1] This priority is respected in the case of default.

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3. Methodology

First of all, we will investigate the convergence conditions of Leland (1994), Goldstein, Ju and Leland (2001) and Sarkar and Zapatero (2003).To achieve this objective, we will use the contingent claims closed form expressions, as obtained by the authors, to identify these conditions analytically. We think that the first two models are just special cases of the third one. We will then compare them separately with this last model. To do so, we will pay special attention to the speed of mean reversion parameter value and use mathematical evidence to obtain similar solutions for contingent claims. In fact, the three models present some similarities but are also different in some respects.

We will also extend Sarkar and Zapatero (2003) by presenting a new computable valuation method for contingent claims, mainly equity and debt, based on financial statements information. We will again use mathematical evidence to obtain finite solutions for contingent claims expressions and then avoid the computation problem caused by the confluent hyper-geometric function present in all the contingent claims expressions of SZ. This will make these contingent claims assessable, which will allow us to determine the company value. Further analyses will help to extend the use of our new model to CEO compensation, mergers and acquisitions, debt cost and cost of capital, stock-options and other assets valuations.

4. Convergence Conditions of Three Structural Models

We examine the Leland (1994), Goldstein, Ju and Leland (2001), and Sarkar and Zapatero (2003) models. We begin by investigating the convergence between the Leland (1994) and Sarkar and Zapatero (2003) models. Then, we check for convergence conditions between the two EBIT models.

4.1. Leland (1994) model vs. Sarkar and Zapatero (2003) model

According to the Leland (1994) model[2], equity value can be expressed as follows:

$$\mathbf{E(V) = V - (1-\tau)\frac{C}{r} + \left[(1-\tau)\frac{C}{r} - V_B\right]\left(\frac{V}{V_B}\right)^{-\frac{2r}{\sigma^2}}} \quad (1)$$

Where:

C represents the consolidated coupon;

σ represents assets volatility;

V represents the company’s assets value;

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[2] According to this model, the dynamic equation fulfilled by a contingent claim can be presented as follows:

$$dV = \mu V dt + \sigma V dz$$

Where:

μ represents the company’s instantaneous growth rate;

σ represents assets volatility;

V represents the company’s assets value;

dz is a Wiener process.

V_B represents the value of a company going bankrupt;

r represents the free rate of return;

τ represents the tax rate;

On the other hand, by solving the dynamic equation of the Sarkar and Zapatero(2003) model[3], we obtain the following expression for equity:

$$\mathbf{E(x) = V - (1-\tau)\frac{C}{r} + \left[(1-\tau)\frac{C}{r} - V_B\right]\left[\frac{M(X)}{M(X_L)}\right]\left(\frac{V-V_2}{V_B-V_2}\right)^{\gamma}} \quad (2)$$

Where:

V represents the company value;

V_B represents the value of a company going bankrupt;

k represents the speed of mean reversion;

r represents the free rate of return;

C represents the bond value;

X represents the EBIT;

X_L represents the EBIT value in the case of bankruptcy.

$$\mathbf{V_1 = \frac{(1-\tau)k\theta}{r(k+r)}};\quad \mathbf{V_2 = \frac{(1-\tau)}{k-r}};$$

$$\mathbf{M(x) = 1 + \frac{a}{b}z + \frac{a(a+1)}{b(b+1)}\frac{z^2}{2!} + \cdots + \frac{a(a+1)\ldots(a+n)}{b(b+1)\ldots(b+n)}\frac{z^n}{n!}};$$

$$\mathbf{z = \frac{2k\theta}{\sigma^2 x}};$$

σ represents the company's assets volatility;

$$\gamma = -a = \frac{1+\frac{2k}{\sigma^2} - \sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}}{2};$$

[3]The dynamic equation of this model can be presented as follows:

$dx = k(\Theta - x)dt + \sigma x dz$

Where:

x represents the EBIT;

k represents the speed of mean reversion;

Θrepresents the mean of earnings before interests and taxes;

Σ represents the assets volatility;

dz is a Wiener process.

$$\mathbf{b} = \mathbf{2} - \mathbf{2} * \frac{1+\frac{2k}{\sigma^2}-\sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2+\frac{8r}{\sigma^2}}}{2} + \frac{2k}{\sigma^2};$$

If we consider the special case where "k" is equal to zero, the term "z" also becomes nil. In this case, the expression of M(x) becomes equal to 1, regardless to the company's solvency level. Note that, under a zero mean reversion speed assumption, the term "V_2"becomesnil as well. By considering this assumption, equation (2) becomes:

$$\mathbf{E(x)} = \mathbf{V} - (\mathbf{1}-\tau)\frac{\mathbf{C}}{\mathbf{r}} + \left[(\mathbf{1}-\tau)\frac{\mathbf{C}}{\mathbf{r}} - \mathbf{V_B}\right]\left(\frac{\mathbf{V}}{\mathbf{V_B}}\right)^{\gamma} \quad (3)$$

We can easily show that in the case of a nil mean reversion speed, the value of "γ" becomes:

$$\gamma = \frac{1-\sqrt{1+\frac{8r}{\sigma^2}}}{2} \quad (4)$$

But when the term "$\frac{8r}{\sigma^2}$" is less than 1, which is a strong assumption for volatile assets, expression "$\sqrt{1+\frac{8r}{\sigma^2}}$"can be approximated by the following term:

$$\left[1+\frac{4r}{\sigma^2}\right] \quad (5)$$

If we insert this result into equation (4), the latter becomes:

$$\gamma = \frac{1-(1+\frac{4r}{\sigma^2})}{2} \quad (6)$$

Finally, by solving equation (6), we obtain the following result:

$$\gamma = -\frac{2r}{\sigma^2} \quad (7)$$

We can then conclude that, under a zero mean reversion speed assumption, the two equity expressions become similar according to Leland (1994) and Sarkar and Zapatero (2003). We will now determine the convergence conditions of Goldstein, Ju and Leland (2001) and Sarkar and Zapatero (2003).

4.2. Goldstein, Ju and Leland (2001) model vs. Sarkar and Zapatero (2003) model

Goldstein, Ju and Leland (2001)[4] obtain the following expression for equity:

$$\mathbf{E(V) = (1-\tau)\left[V + \left(\frac{V}{V_B}\right)^{-x}\left(\frac{C}{r} - V_B\right) - \frac{C}{r}\right]} \quad (8)$$

Where:

V represents the value of the company's assets;

V_B represents the value of the company going bankrupt;

C represents the consolidated coupon;

r represents the free rate of return;

τ represents the tax rate;

$$x = \frac{1}{\sigma^2}\left[\left(\mu - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2}\right]$$

σ represents the volatility of returns.

If we consider risk neutrality, we can approximate μ as follows:

$$\mu \approx r$$

This hypothesis allows us to express the term *"x"* as follows:

$$\mathbf{x = \frac{1}{\sigma^2}\left[\left(r - \frac{1}{2}\sigma^2\right) + \sqrt{\left(r - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2}\right]} \quad (9)$$

Equation (9) is equivalent to equation (10) which can be presented as follows:

$$\mathbf{x = \frac{2r}{\sigma^2}} \quad (10)$$

By inserting these results into GJL's equity equation, its expression becomes identical to the one of SZ. The results obtained in Section IV show that Leland (1994) and GJL (2001) are just special cases of SZ (2003) and that the log-normality assumption is pertinent solely in the absence of mean reversion. That being said, all the contingent claims expressions obtained by this last model present a confluent hyper-geometric function that makes their computability

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[4] The dynamic equation of this model can be presented as follows:

$$\frac{dV + \delta dt}{V} = rdt + \sigma dz$$

Where:

V represents the value of the company's assets

δ represents the EBIT;

σ represents the volatility of returns;

dz is a Wiener process.

unsuitable. In Section V, we present a new valuation technique based on finite solutions.

5. A New Capital-Structure-Based Valuation Method

The Sarkar and Zapatero (2003) model is a sophisticated one but cannot be operationalized for contingent claims evaluation. In this section, we extend Sarkar and Zapatero's (2003) theory by presenting a new valuation method based on usual computable functions. We will proceed by conducting a convergence analysis. So, we know that the term "M(X)" has a minimum value of 1. We also know that the term "γ" can be presented as follows:

$$\gamma = \frac{\left(1+\frac{2k}{\sigma^2}\right) - \sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2 + \left(\frac{8r}{\sigma^2}\right)}}{2} \qquad (11)$$

We also know that the parameter *"b"* of *"M(x)"* can be expressed as follows:

$$b = 2 - 2x\frac{\left(1+\frac{2k}{\sigma^2}\right) - \sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2 + \left(\frac{8r}{\sigma^2}\right)}}{2} + \frac{2k}{\sigma^2} \qquad (12)$$

By developing this expression, we obtain the following:

$$b = 1 + \sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2 + \left(\frac{8r}{\sigma^2}\right)} \qquad (13)$$

So, we can easily demonstrate that *"-γ = a"* never exceeds the value of *"b"*[5]. We also know that the nth term of *"M(x)"* can be expressed as follows:

$$\frac{-\gamma(-\gamma+1)\ldots(-\gamma+n)}{b(b+1)\ldots(b+n)} \qquad (14)$$

Thus, when n tends to infinity, meaning that the company remains solvent, the expression $\frac{(-\gamma+n)}{(b+n)}$ tends to 1, representing an upper boundary. We can then approximate *"M(x)"* as follows:

$$M(a;b;z) \to 1 + z + \frac{z^2}{2!} + \cdots + \frac{z^n}{n!} \qquad (15)$$

Equation (15) represents the Taylor series expansion of an exponential function. Thus, we can say that the confluent hyper-geometric function *"M(x)"* tends to the expression below:

$$M(x) \to e^z = e^{\frac{2k\theta}{\sigma^2 x}} \qquad (16)$$

[5] $b = 1 + \sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2 + \left(\frac{8r}{\sigma^2}\right)} \geq \sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2 + \left(\frac{8r}{\sigma^2}\right)} \geq \sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2 + \left(\frac{8r}{\sigma^2}\right)} - \left(1+\frac{2k}{\sigma^2}\right) \geq \frac{\sqrt{\left(1+\frac{2k}{\sigma^2}\right)^2 + \left(\frac{8r}{\sigma^2}\right)} - \left(1+\frac{2k}{\sigma^2}\right)}{2} = -\gamma$

Since *"k"*, *"θ"* and *"σ"* are finite terms, *"M(x)"* obviously converges when *"x"* becomes very large. This allows us to express equity, in the case of solvency, as follows:

$$\mathbf{E(x) = V - (1-\tau)\frac{C}{r} + \left[(1-\tau)\frac{C}{r} - V_B\right]\left[\exp\left(\frac{2k\theta V_1}{\sigma^2}\right)\left(\frac{1}{V-V_2} - \frac{1}{V_B-V_2}\right)\right]\left(\frac{V-V_2}{V_B-V_2}\right)^{\gamma}}$$

(17)

This expression represents a finite function that is easy to compute. We can then use it to determine the fair value of equity.

6. Conclusion

One of the main purposes of this paper was to investigate conditions under which three structural models became similar. The models we compared are Leland (1994), Goldstein, Ju and Leland (2001), and Sarkar and Zapatero (2003). The three models have divergent dynamic processes and non-identical state variables. The analysis showed that the first two models were just special cases of the last one. We also established that the log-normality hypothesis held just in the case of a zero-mean-reversion-speed.

Also, we presented a convenient stakeholders' shares evaluation method, based on financial statements information. Further investigations will allow us to measure the performance of such a technique in evaluating a company's fair value. The new evaluation method we presented is equally useful for several other purposes like debt cost and cost of capital estimation, CEO'S compensation, stock-options fair value determination and mergers and acquisitions evaluation. Future empirical analyses using this model should be performed to show the accounting information power in determining the proper value of company assets.

That being said, the model presents some limits. In fact, it is only useful for mean-reversion companies. Furthermore, it can only be operationalized for companies with a positive EBIT and a debt value lower than the company value as calculated by the model itself. The opposite means that the company suffers financial distress.The model represents a first step towards avoiding the market inefficiency while measuring an asset value. Further structural models presenting lower restrictions will be performed to determine a company's fair value.

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