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Inevitability of Budget Deficit in a Growing Economy

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Abstract

Using a simple macroeconomic model and a model with microeconomic foundations about behavior of consumers and firms, we examine the budget deficit in a growing economy, and will show mainly the following results. 1) If the initial savings is positive (or investment of private firms is not large), we need budget deficit including interest payments on government bonds to maintain full employment with or without inflation. 2) If the initial savings is negative (or investment of private firms is large), investment of private firms cannot be financed only by consumers' investment, but also by the government. 3) If the interest rate on the government bonds is larger than the growth rate, the budget surplus excluding interest payments is necessary to maintain full employment under constant prices.

Keywords: inevitability of budget deficit, growing economy, MMT, Functional Finance Theory

1. Introduction

In this paper we examine the role of budget deficit in an economy which grows by technological progress from the perspective of the Functional Finance Theory (Lerner, 1943, 1944) and MMT (Modern Monetary Theory or Modern Money Theory, Wray, 2015; Kelton, 2020; Mitchell, Wray, & Watts, 2019, Note 1). We consider two types of the model of a growing economy. The first is a simple macroeconomic model which includes investment of firms. However, there is no microeconomic foundation about behaviors of consumers and firms. The second is a model with microeconomic foundations of consumers and firms with bequest motive of consumers without investment of firms. The firms produce differentiated goods under monopolistic competition.

In the next section we examine the budget deficit in a simple macroeconomic model with investment of private firms, and will show the following results.

1) (Proposition 1)

(i) If the initial savings is positive (or investment of private firms is not large), we need budget deficit including interest payments on government bonds to maintain full employment with or without inflation.

(ii) If the budget deficit is larger than the value, which is necessary and sufficient to maintain full employment without inflation in eq. (4), inflation is induced, and it is smaller than that value, recession is triggered.

(iii) If the interest rate of government bonds is larger than the real growth rate, we need budget surplus excluding interest payments to maintain full employment without inflation.

2) (Proposition 2)

(i) If the initial savings is negative (or investment of private firms is large), we need budget surplus including interest payments to maintain full employment with or without inflation.

(ii) If the initial savings is negative, investment of private firms cannot be financed only by consumers' investment, but also by the government.

In Section 3 we use a model with microeconomic foundations about behaviors of consumers and firms with bequest motive of consumers, and will show the following results.

1) (Proposition 3)

(i) We need budget deficit (including interest payments on government bonds) to maintain full employment when the economy grows at the positive rate by technological progress under constant prices.

(ii) If the interest rate on the government bonds is larger than the growth rate, the budget surplus excluding interest payments is necessary to maintain full employment under constant prices.

2) (Proposition 4)

(1) If the budget deficit is larger than the level which is necessary and sufficient to maintain full employment under constant prices, inflation is triggered.

(2) If inflation occurs for one period only, then the steady-state value of each variable multiplied by the inflation rate returns the economy to growth under constant prices.

(3) If inflation continues, then growth, including price increases, continues under the value of each variable that caused inflation multiplied by the inflation rate.

2. A Simple Macroeconomic Model with Investment

In this section we use a simple macroeconomic model with investment of private firms. We assume full employment. Let Y_1, I_1, G_1 be GDP, investment and fiscal spending in a period (Period 1), S_0 be the savings carried over from the previous period (Period 0), $c,\ (0<c<1)$, $t,\ (0<t<1)$, $r,\ (0<r<1)$ be the propensity to consume of the consumers, the tax rate and the interest rate of government bonds. In Period 1 we have

$$Y_1 = c(1-t)Y_1 + c(1+r)S_0 + I_1 + G_1.$$

The consumers spend the portion of the savings, including interest, carried over from Period 0 and income, represented by c, on consumption. The savings in Period 1 is obtained as follows.

$$S_1 = (1-c)(1+r)S_0 + (1-c)(1-t)Y_1 - I_1 = G_1 - tY_1 + (1+r)S_0.$$

The savings are made in the form of government bonds. They do not include purchase of stocks. It is an investment I_1. Then, we get

$$S_1 - S_0 = G_1 - tY_1 + rS_0. \tag{1}$$

This is the difference between the savings in Period 1 and that in Period 0. (1) means that it equals the budget deficit including interest payments on the government bonds.

Similarly, in the next period, Period 2, we have

$$Y_2 = c(1-t)Y_2 + c(1+r)S_1 + I_2 + G_2.$$

Y_2, I_2, G_2 be GDP, investment and fiscal spending in Period 2. They are nominal values. Suppose the economy is growing nominally at a rate of $g+p+gp$ from one period to the next. g is the real growth rate, and p is the inflation rate. Then, the nominal growth rate is

$$(1+g)(1+p)-1=g+p+gp.$$

We can assume

$$Y_2=(1+g)(1+p)Y_1,\ I_2=(1+g)(1+p),$$

and we obtain

$$(1+g)(1+p)Y_1 \\ =(1+g)(1+p)c(1-t)Y_1+c(1+r)S_1+(1+g)(1+p)I_1+(1+g)(1+p)G_1.$$

The savings in Period 2 is

$$S_2=(1-c)(1+r)S_1+(1+g)(1+p)(1-c)(1-t)Y_1-(1+g)(1+p)I_1$$

$$=(1+g)(1+p)G_1-(1+g)(1+p)tY_1+(1+r)S_1.$$

Therefore, we find

$$S_2-S_1=G_2-tY_2+rS_1=(1+g)(1+p)G_1-(1+g)(1+p)tY_1+rS_1.$$

Generally, for Period n,

$$(1+g)^{n-1}(1+p)^{n-1}Y_1 \\ =(1+g)^{n-1}(1+p)^{n-1}c(1-t)Y_1+c(1+r)S_{n-1}+(1+g)^{n-1}(1+p)^{n-1}I_1+(1 \\ +g)^{n-1}(1+p)^{n-1}G_1.$$

The savings in Period n is

$$S_n=(1-c)(1+r)S_{n-1}+(1+g)^{n-1}(1+p)^{n-1}(1-c)(1-t)Y_1-(1+g)^{n-1}(1+p)^{n-1}I_1$$

$$=(1+g)^{n-1}(1+p)^{n-1}G_1-(1+g)^{n-1}(1+p)^{n-1}tY_1+(1+r)S_{n-1}.$$

Therefore,

$$S_n-S_{n-1}=(1+g)^{n-1}(1+p)^{n-1}G_1-(1+g)^{n}(1+p)^{n}tY_1+rS_{n-1} \quad (2)$$

$$= G_n - tY_n + rS_{n-1}.$$

This means that the difference between the savings in Period n and that in Period $n-1$ equals the budget deficit including interest payments on government bonds. The following equation would hold for a steady-state growth path.

$$S_n = (1+g)^n(1+p)^n S_0,\ S_{n-1} = (1+g)^{n-1}(1+p)^{n-1}S_0.$$

Then, (2) means

$$(1+g)^{n-1}(1+p)^{n-1}(g+p+gp)S_0 = (1+g)^{n-1}(1+p)^{n-1}(G_1 - tY_1 + rS_0).$$

Thus, we have

$$G_1 - tY_1 + rS_0 = (g+p+gp)S_0,$$

or

$$G_1 - tY_1 = (g+p+gp-r)S_0.$$

Generally, for Period n,

$$G_n - tY_n + rS_{n-1} = (1+g)^{n-1}(g+p+gp)S_0, \tag{3}$$

or

$$G_n - tY_n = (1+g)^{n-1}(g+p+gp-r)S_0.$$

Now assume $S_0 > 0$, that is, the savings in Period 0 is positive. We call it the initial savings. In this case investment of private firms is not large. (3) is positive whether or not $p > 0$ or $p = 0$, that is, with or without inflation. Thus, we need budget deficit to maintain full employment with or without inflation. Since we assume full employment, assuming $p = 0$ in (3), the budget deficit which is necessary and sufficient to achieve full employment without inflation is obtained as follows.

$$G_n - tY_n + rS_{n-1} = (1+g)^{n-1}gS_0. \tag{4}$$

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If the budget deficit is larger than this value, inflation is induced, and if it is smaller than this value, recession is triggered.

From (3)

$$G_n - tY_n = (1+g)^{n-1}(g-r)S_0.$$

Therefore, if $r > g$, that is, the interest rate of government bonds is larger than the real growth rate, we need budget surplus excluding interest payments $(G_n - tY_n < 0)$ to maintain full employment without inflation. Summarizing the results,

Proposition 1

1) If the initial savings is positive (or investment of private firms is not large), we need budget deficit including interest payments on government bonds to maintain full employment with or without inflation.

2) If the budget deficit is larger than the value, which is necessary and sufficient to maintain without inflation in (4), inflation is induced, and it is smaller than the value in (4), recession is triggered.

3) If the interest rate of government bonds is larger than the real growth rate, we need budget surplus excluding interest payments to maintain full employment without inflation.

Now consider a case where $S_0 < 0$, that is, the initial savings is negative. Then,

$$S_1 = (1+g)(1+p)S_0 < 0,$$

and

$$S_n = (1+g)^n(1+p)^n S_0 < 0.$$

Therefore, the savings in each period is negative. In this case

$$tY_1 - G_1 - rS_0 = -(g+p+gp)S_0, > 0,$$

and

$$tY_n - G_n - rS_{n-1} = -(1+g)^{n-1}(g+p+gp)S_0 > 0.$$

These mean that we need budget surplus including interest payments to maintain full employment with or without inflation. Negative savings implies the following:

$$I_1 > (1-c)(1+r)S_0 + (1-c)(1-t)Y_1,$$

$$I_n > (1-c)(1+r)S_{n-1} + (1-c)(1-t)Y_n,$$

$$I_1 = (1-c)(1+r)S_0 + (1-c)(1-t)Y_1 + tY_1 - G_1 - rS_0,$$

and

$$I_n = (1-c)(1+r)S_{n-1} + (1-c)(1-t)Y_n + tY_n - G_n - rS_{n-1},$$

for each period. Therefore, investment of private firms cannot be financed only by consumers' investment (stock purchase), but also by the government. This is the meaning of the budget surplus. We have shown the following results

Proposition 2

1) If the initial savings is negative (or investment of private firms is large), we need budget surplus including interest payments to maintain full employment with or without inflation.

2) If the initial savings is negative, investment of private firms cannot be financed only by consumers' investment, but also by the government.

3. A Model with Microeconomic Foundations Without Investment

3.1 The Model

Next we consider a model with microeconomic foundations under monopolistic competition. Our model is a two-periods (1: younger or working, and 2: older or retired) overlapping generations (OLG) model. It is according to Otaki (2007, 2009, 2015), and a generalization of Tanaka (2020) in which perfect competition is assumed. The structure of our model is as follows.

1) There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0,1]$. Good z is monopolistically produced by firm z with increasing or decreasing or constant returns to scale technology. The technology progresses at the rate $g > 0$.

2) During Period 1, each consumer supplies labor, consumes the goods and saves asset and income for his consumption and bequest in Period 2. He is employed or not employed. Savings are made through government bonds that earn interest. More generally, savings can be thought of as being made in terms of interest-bearing government bonds and non-interest-bearing money with endogenously determined interest rate of government bonds, but the results of the analysis are similar.

3) During Period 2, each consumer consumes the goods using his savings carried over from his Period 1, and he leaves a bequest to the next generation. We assume that the bequest is spent by the next generation consumers in their Period 1, that is, it is spent in Period 2 of the older generation consumers. Therefore, the bequest is not a literal bequest but a gift from the older generation to the younger generation, since it is not used after the death of the previous generation. The bequest is equally distributed to each younger consumer.

4) Each consumer determines his consumptions and bequest in Periods 1 and 2 and the labor supply at the beginning of Period 1 depending on the situation that he is employed or not employed.

We use the following notation.

C_i^e: consumption basket of an employed consumer in Period i, $i = 1,2$.

B^e: Bequest by an employed consumer.

C_i^u: consumption basket of an unemployed consumer in Period i, $i = 1,2$.

B^u: Bequest by an unemployed consumer.

$c_i^e(z)$: consumption of good z of an employed consumer in Period i, $i = 1,2$.

$c_i^u(z)$: consumption of good z of an unemployed consumer in Period i, $i = 1,2$.

P_i: the price of consumption basket in Period i, $i = 1,2$.

$p_i(z)$: the price of good z in Period i, $i = 1,2$.

$p + 1 = P_2/P_1$: (expected) inflation rate (plus one).

W: nominal wage rate.

Π: profits of firms which are equally distributed to each consumer.

r: interest rate of government bonds.

l: labor supply of an individual.

$\Gamma(l)$: disutility function of labor, which is increasing and convex.

L: total employment.

L_f: population of labor or employment in the full employment state.

y: labor productivity, which increases by technological progress, also it is increasing or decreasing or constant with respect to the total employment, Ll.

g: economic growth rate, $g > 0$.

We assume that the population L_f is constant.

We denote a bequest to each consumer by consumers of the previous generation by $\tilde{B}$. We have

$$\tilde{B} = \frac{1}{L_f}[L\tilde{B}^e + (L_f - L)\tilde{B}^u],\ L_f\tilde{B} = L\tilde{B}^e + (L_f - L)\tilde{B}^u.$$

$\tilde{B}^e$ and $\tilde{B}^u$ are, respectively, bequests by an employed consumer and an unemployed consumer of the previous generation.

3.2 Consumers' Behavior

First we consider utility maximization of consumers. The utility function of employed consumers of one generation over two periods is written as

$$u(C_1^e, C_2^e, \frac{B^e}{P_2}) - \Gamma(l).$$

The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, \frac{B^u}{P_2}).$$

We assume that $u(\cdot,\cdot,\cdot)$ is a homothetic function. The consumption baskets of employed and unemployed consumers in Period i are

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$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, i = 1,2,$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, i = 1,2.$$

σ is the elasticity of substitution among the goods. $\sigma > 1$.

The price of consumption basket in Period i is defined by

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}}, i = 1,2.$$

The budget constraint for an employed consumer is

$$\int_0^1 p_1(z)c_1^e(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2^e(z)dz + \frac{1}{1+r}B^e = Wl + \Pi + (1+r)\tilde{B}.$$

The budget constraint for an unemployed consumer is

$$\int_0^1 p_1(z)c_1^u(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2^u(z)dz + \frac{1}{1+r}B^u = \Pi + (1+r)\tilde{B}.$$

Let

$$\alpha = \frac{P_1C_1^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + \frac{1}{1+r}B^e}, \beta = \frac{\frac{1}{1+r}P_2C_2^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + \frac{1}{1+r}B^e}, \ 1-\alpha-\beta = \frac{\frac{1}{1+r}B^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + B^e}.$$

$0 < \alpha < 1$, $0 < \beta < 1$. Since the utility functions $u(C_1^e, C_2^e, \frac{B^e}{P_2})$ and $u(C_1^u, C_2^u, \frac{B^u}{P_2})$are homothetic, α and β are determined by the prices, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1C_1^e}{P_1C_1^e + \frac{1}{1+r}P_2C_2^e + \frac{1}{1+r}B^e} = \frac{P_1C_1^u}{P_1C_1^u + \frac{1}{1+r}P_2C_2^u + \frac{1}{1+r}B^u},$$

$$\beta = \frac{\frac{1}{1+r}P_2C_2^e}{P_1C_1^e+\frac{1}{1+r}P_2C_2^e+\frac{1}{1+r}B^e} = \frac{\frac{1}{1+r}P_2C_2^u}{P_1C_1^u+\frac{1}{1+r}P_2C_2^u+\frac{1}{1+r}B^u},$$

and

$$1-\alpha-\beta = \frac{B^e}{(1+r)P_1C_1^e+P_2C_2^e+B^e} = \frac{B^u}{(1+r)P_1C_1^u+P_2C_2^u+B^u}.$$

From standard calculations we obtain the following demand functions for consumption baskets and bequests (please see Appendix A).

$$C_1^e = \alpha\frac{Wl+\Pi+(1+r)\tilde{B}}{P_1},\ C_2^e = \beta\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2},\ \frac{B_2^e}{P_2} = (1-\alpha-\beta)\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2},$$

$$C_1^u = \alpha\frac{\Pi+(1+r)\tilde{B}}{P_1},\ C_2^u = \beta\frac{\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2},\ \frac{B^u}{P_2} = (1-\alpha-\beta)\frac{\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}.$$

Also the following demand functions for good z of employed and unemployed consumers are derived.

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi+(1+r)\tilde{B})}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{\beta(Wl+\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2},$$

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(\Pi+(1+r)\tilde{B})}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{\beta(\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e = u\left(\alpha\frac{Wl+\Pi+(1+r)\tilde{B}}{P_1}, \beta\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}, (1-\alpha-\beta)\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}\right) - \Gamma(l),$$

and

$$V^u = u\left(\alpha\frac{\Pi+(1+r)\tilde{B}}{P_1}, \beta\frac{\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}, (1-\alpha-\beta)\frac{Wl+\Pi+(1+r)\tilde{B}}{\frac{1}{1+r}P_2}\right).$$

Let

$$\omega = \frac{W}{P_1}.$$

This is the real wage rate. Then, we can write

$$V^e = \varphi\left(\omega l + \frac{\Pi+(1+r)\tilde{B}}{P_1}, p, 1+r\right) - \Gamma(l),$$

$$V^u = \varphi\left(\frac{\Pi+(1+r)\tilde{B}}{P_1}, p, 1+r\right),$$

Denote

$$I = \omega l + \frac{\Pi+(1+r)\tilde{B}}{P_1}.$$

The condition for maximization of V^e with respect to l given p is

$$\frac{\partial\varphi}{\partial I}\omega - \Gamma'(l) = 0,$$

where

$$\frac{\partial\varphi}{\partial I} = \alpha\frac{\partial u}{\partial c_1^e} + \beta\left(\frac{1+r}{1+p}\right)\frac{\partial u}{\partial c_2^e} + (1-\alpha-\beta)\left(\frac{1+r}{1+p}\right)\frac{\partial u}{\partial\left(\frac{B^e}{P_2}\right)}.$$

Given P_1, p and r the labor supply is a function of ω.

3.3 Firms' Behavior and the Government

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl+L_f\Pi+L_f(1+r)\tilde{B})}{P_1} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl+L_f\Pi+L_f(1+r)\tilde{B})}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{\beta(WLl+L_f\Pi+L_f(1+r)\tilde{B})}{\frac{1}{1+r}P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\beta(\overline{W}\ \overline{L}\overline{l}+L_f\overline{\Pi}+L_f(1+r)\overline{\tilde{B}})}{\frac{1}{1+r}P_1},$$

where $\overline{W}$, $\overline{\Pi}$, $\overline{L}$, $\overline{l}$ and $\overline{\tilde{B}}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the bequest from the previous generation, respectively, during the previous period. In the equilibrium all $p_1(z)$ are equal, and all $p_2(z)$ are equal.

Let

$$M = \beta\left(\overline{W}\ \overline{L}\overline{l} + L_f\overline{\Pi} + L_f(1+r)\overline{\tilde{B}}\right).$$

The total consumption of the older generation consumers is

$$(1+r)M = \beta(1+r)\left(\overline{W}\ \overline{L}\overline{l} + L_f\overline{\Pi} + L_f(1+r)\overline{\tilde{B}}\right).$$

It is the planned consumption that is determined in their Period 1. Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{M}{P_1}$. The total savings (with interest) is

$$(1-\alpha)(1+r)\left(\overline{W}\ \overline{L}\overline{l} + L_f\overline{\Pi} + L_f(1+r)\overline{\tilde{B}}\right).$$

Government expenditure constitutes the national income as well as the consumptions of the

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younger and older generations. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{Y}{P_1}, \tag{5}$$

where Y is the effective demand defined by

$$Y = \alpha(WLl + L_f\Pi + L_f(1+r)\tilde{B}) + G + (1+r)M.$$

G is the government expenditure. The government determines its demand for good z, $g(z)$, to maximize the following index.

$$\left(\int_0^1 g(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},$$

subject to the constraint:

$$\int_0^1 p_1(z)g(z)dz = G.$$

Let L and Ll be employment and the "employment $\times$ labor supply" of firm z. The total employment and the total "employment $\times$ labor supply" are

$$\int_0^1 Ldz = L,\ \int_0^1 Lldz = Ll.$$

The output of firm z is Lly. By increasing or decreasing returns to scale y is a function of Ll. In the equilibrium $Lly = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial Ll} = y + Lly'(Ll).$$

In a case of constant returns to scale

$$\frac{\partial d(z)}{\partial Ll} = y.$$

From (5)

$$\frac{\partial p_1(z)}{\partial d(z)} = -\frac{p_1(z)}{\sigma d(z)}.$$

Thus,

$$\frac{\partial p_1(z)}{\partial Ll} = -\frac{p_1(z)[y+Lly'(Ll)]}{\sigma d(z)} = -\frac{p_1(z)}{\sigma Ll}\left[1+\frac{Lly'(Ll)}{y}\right].$$

Define the elasticity of the labor productivity by

$$\zeta = \frac{Lly'(Ll)}{y}.$$

Then,

$$\frac{\partial p_1(z)}{\partial Ll} = -\frac{p_1(z)(1+\zeta)}{\sigma Ll}$$

We assume that ζ is constant and $1+\zeta>0$. For increasing (or decreasing) returns to scale technology $\zeta>0$ (or $(\zeta<0)$).

The profit of firm z is

$$\pi(z) = p_1(z)Lly - LlW.$$

The condition for profit maximization is

$$\frac{\partial \pi(z)}{\partial Ll} = \left[p_1(z)y - Lly\frac{p_1(z)}{\sigma Ll}\right](1+\zeta) - W = \left[p_1(z)y - \frac{p_1(z)y}{\sigma}\right](1+\zeta) - W = 0.$$

Therefore, we obtain

$$p_1(z) = \frac{1}{\left(1-\frac{1}{\sigma}\right)(1+\zeta)y}W.$$

Let $\mu = 1/\sigma$. Then,

$$p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y}W.$$

This means that the real wage rate is

$$\omega = (1-\mu)(1+\zeta)y.$$

Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y}W.$$

3.4 Market Equilibrium

The (nominal) aggregate supply of the goods is equal to

$$WL + L_f\Pi = P_1Lly.$$

The (nominal) aggregate demand is

$$\alpha(WL + L_f\Pi + L_f(1+r)\tilde{B}) + G + (1+r)M = \alpha P_1Lly + \alpha L_f(1+r)\tilde{B} + G + (1+r)M.$$

Since they are equal,

$$P_1Lly = \alpha P_1Lly + \alpha L_f(1+r)\tilde{B} + G + (1+r)M. \quad (6)$$

In real terms

$$Lly = \frac{G+(1+r)M+\alpha L_f(1+r)\tilde{B}}{(1-\alpha)P_1}.$$

The equilibrium value of Ll cannot be larger than $L_fl(L_f)$. $l(L_f)$ is the labor supply when full employment is achieved. However, Ll may be strictly smaller than $L_fl(L_f)$. Then, we have $L < L_f$ and involuntary unemployment exists. If the government collects a tax T from the younger generation consumers, (6) is rewritten as

$$P_1Lly = \alpha(P_1Lly + L_f(1+r)\tilde{B} - T) + G + M.$$

3.5 Budget Deficit to Maintain Full Employment

Suppose that up to Period t full employment has been achieved under constant prices. Then, the following equation holds.

$$P_1^t L_f l(L_f)y = \alpha(P_1^t L_f l(L_f)y + L_f(1+r)\tilde{B}^{t-1} - T^t) + G^t + (1+r)M^{t-1}. \quad (7)$$

Superscript t represents the values in Period t. M^{t-1} is the savings, and $\tilde{B}^{t-1}$ is the bequest of the previous generation consumers. The savings of the younger generation consumers is

$$M^t + L_f\tilde{B}^t = (1-\alpha)\left(P_1^t L_f l\left(L_f\right)y + L_f(1+r)\tilde{B}^{t-1} - T^t\right) \quad (8)$$

$$= G^t + L_f(1+r)\tilde{B}^{t-1} - T^t + (1+r)M^{t-1}.$$

Their consumptions in their Period 2 and the bequests are, respectively,

$$\beta(1+r)(P_1^t L_f l(L_f)y + L_f(1+r)\tilde{B}^{t-1} - T^t)$$
$$= \frac{\beta}{1-\alpha}(1+r)[G^t + L_f(1+r)\tilde{B}^{t-1} - T^t + (1+r)M^{t-1}],$$

and

$$(1-\alpha-\beta)(1+r)(P_1^t L_f l(L_f)y + L_f(1+r)\tilde{B}^{t-1} - T^t)$$
$$= \frac{1-\alpha-\beta}{1-\alpha}(1+r)[G^t + L_f(1+r)\tilde{B}^{t-1} - T^t + (1+r)M^{t-1}].$$

In order to maintain full employment under growth by technological progress (8) must be equal to $(1+g)(M^{t-1} + L_f\tilde{B}^{t-1})$. Therefore, we obtain

$$G^t - T^t = (g-r)(M^t + L_f\tilde{B}^{t-1}),$$

or

$$G^t - T^t + r(M^{t-1} + L_f\tilde{B}^{t-1}) = g(M^{t-1} + L_f\tilde{B}^{t-1}). \quad (9)$$

The left hand-side is the budget deficit including interest payments on government bonds. Since $M^{t-1} + L_f\tilde{B}^{t-1}$ is positive, $G^t - T^t + r(M^{t-1} + L_f\tilde{B}^{t-1}) > 0$ when $g > 0$. In Period $t+1$ $M^t = (1+g)M^{t-1}$, $\tilde{B}^t = (1+g)\tilde{B}^{t-1}$, and we can assume $G^{t+1} = (1+g)G^t$ and $T^{t+1} = (1+g)T^t$. Thus, with $P_1^{t+1} = P_1^t$ we obtain

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$P_1^t L_f l(L_f)(1+g)y = \alpha(P_1^t L_f l(L_f)(1+g)y - (1+g)T^t + (1+g)L_f(1+r)\tilde{B}^{t-1}) + (1+g)G^t + (1+g)(1+r)M^{t-1}$.

This is equivalent to (7), and full employment is maintained by $G^{t+1} = (1+g)G^t$ and $T^{t+1} = (1+g)T^t$.

Budget deficit is necessary under growth because of deficiency of the savings of the older generation. (9) means that an increase in the savings from Period t to the next period equals the budget deficit including interest payments on government bonds. (9) does not holds when the left-hand side is zero or negative, that is, under budget surplus or balanced budget including interest payments full employment is not realized in a growing economy with constant prices. However, if $r > g$, that is the interest rate on government bonds is larger than the growth rate, we need budget surplus $(G^t - T^t < 0)$ excluding interest payments. Summarizing the results.

Proposition 3

(1) We need budget deficit (including interest payments on government bonds) to maintain full employment when the economy grows at the positive rate by technological progress under constant prices.

(2) If the interest rate on the government bonds is larger than the growth rate, the budget surplus excluding interest payments is necessary to maintain full employment under constant prices.

3.6 Excessive Budget Deficit and Inflation

Suppose that up to Period $t-1$ full employment has been achieved under constant prices, but the government expenditure and/or the tax in Period t may be different from their steady state values. The steady state is a state such that full employment is continuously maintained under constant prices. Denote them by $\hat{G}^t$ and $\hat{T}^t$. We denote also the actual price by $\hat{P}_1^t$. Then, the following equation holds.

$$\hat{P}_1^t L_f l(L_f)y = \alpha(\hat{P}_1^t L_f l(L_f)y + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t) + \hat{G}^t + (1+r)M^{t-1}. \quad (10)$$

The savings of the younger generation (with interest) is

$$M^t + L_f(1+r)\tilde{B}^t = (1-\alpha)(\hat{P}_1^t L_f l(L_f)y + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t) \quad (11)$$
$$= \hat{G}^t + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t + (1+r)M^{t-1}.$$

Their consumptions in their Period 2 and the bequests are, respectively,

$$\beta(1+r)(\hat{P}_1^t L_f l(L_f)y + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t)$$
$$= \frac{\beta}{1-\alpha}(1+r)(\hat{G}^t + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t + (1+r)M^{t-1}),$$

and

$$(1-\alpha-\beta)(1+r)(\hat{P}_1^t L_f l(L_f)y + L_f\tilde{B}^{t-1} - \hat{T}^t)$$
$$= \frac{1-\alpha-\beta}{1-\alpha}(1+r)(\hat{G}^t + L_f(1+r)\tilde{B}^{t-1} - \hat{T}^t + (1+r)M^{t-1}).$$

Let

$$1 + p = \frac{\hat{P}_1^t}{P_1^t} > 1.$$

If in Period $t+1$ full employment is maintained with $P_1^{t+1} = \hat{P}_1^t > P_1^t$, (11) must be equal to $(1+g)(1+p)(M^{t-1} + L_f\tilde{B}^{t-1})$. Thus,

$$\hat{G}^t - \hat{T}^t = (g + p + gp - r)(M^{t-1} + L_f\tilde{B}^{t-1}),$$

or

$$\hat{G}^t - \hat{T}^t + r(M^{t-1} + L_f\tilde{B}^{t-1}) = (g + p + gp)(M^{t-1} + L_f\tilde{B}^{t-1}).$$

Then, from (9)

$$\hat{G}^t - \hat{T}^t + r(M^{t-1} + L_f\tilde{B}^{t-1}) = (g + p + gp)(M^{t-1} + L_f\tilde{B}^{t-1}) > g(M^{t-1} + L_f\tilde{B}^{t-1})$$
$$= G^t - T^t + r(M^{t-1} + L_f\tilde{B}^{t-1}).$$

This means

$$(1+g)p(M^{t-1} + L_f\tilde{B}^{t-1}) = \hat{G}^t - \hat{T}^t - (G^t - T^t),$$

or

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$$p = \frac{\hat{G}^t - \hat{T}^t - (G^t - T^t)}{(1+g)(M^{t-1} + L_f \tilde{B}^{t-1})} > 0.$$

Therefore, excessive budget deficit, $\hat{G}^t - \hat{T}^t - (G^t - T^t)$, causes inflation at the rate of $\frac{\hat{P}_1^t}{P_1^t} - 1$.

In Period $t+1$, $M^{t+1} = (1+g)(1+p)M^t$, $\tilde{B}^{t+1} = (1+g)(1+p)\tilde{B}^t$, and we assume $G^{t+1} = (1+g)(1+p)G^t$ and $T^{t+1} = (1+g)(1+p)T^t$. Thus, with $P_1^{t+1} = \hat{P}_1^t$ we obtain

$$\hat{P}_1^t L_f l(L_f)(1+g)y = \alpha(\hat{P}_1^t L_f l(L_f)(1+g)y + (1+g)(1+p)L_f(1+r)\tilde{B}^{t-1} - (1+g)(1+p)T^t) + (1+g)(1+p)G^t + (1+g)(1+p)(1+r)M^{t-1}.$$

Since $\hat{P}_1^t = (1+p)P_1^t$, this is equivalent to (7), and full employment is maintained by $G^{t+1} = (1+g)(1+p)G^t$ and $T^{t+1} = (1+g)(1+p)T^t$. In this case, inflation occurs for one period only, and the steady-state value of each variable multiplied by the inflation rate returns the economy to growth under constant prices.

On the other hand, suppose that in Period $t+1$ $P_1^{t+1} = (1+p)\hat{P}_1^t$, that is, inflation continues. Then, the following equation holds.

$$\hat{P}_1^t L_f l(L_f)(1+g)(1+p)y = \alpha(\hat{P}_1^t L_f l(L_f)(1+g)(1+p)y + (1+g)(1+p)L_f(1+r)\tilde{B}^{t-1} - (1+g)(1+p)\hat{T}^t) + (1+g)(1+p)\hat{G}^t + (1+g)(1+p)(1+r)M^{t-1}.$$

This is equivalent to (10), and full employment is maintained . In this case, inflation continues and growth, including price increases, continues under the value of the variables that caused inflation multiplied by the inflation rate.

Summarizing the results.

Proposition 4

(1) If the budget deficit is larger than the level which is necessary and sufficient to maintain full employment under constant prices, inflation is triggered.

(2) If inflation occurs for one period only, then the steady-state value of each variable multiplied by the inflation rate returns the economy to growth under constant prices.

(3) If inflation continues, then growth, including price increases, continues under the value of each variable that caused inflation multiplied by the inflation rate.

4. Conclusion

Using a simple macroeconomic model and a model with microeconomic foundations about behaviors of consumers and firms, we have examined the budget deficit in a growing economy, and mainly have shown the following results.

1) If the initial savings is positive (or investment of private firms is not large), we need budget deficit including interest payments on government bonds to maintain full employment with or without inflation.

2) If the initial savings is negative (or investment of private firms is large), investment of private firms cannot be financed only by consumers' investment, but also by the government.

3) If the interest rate on the government bonds is larger than the growth rate, the budget surplus excluding interest payments is necessary to maintain full employment under constant prices.

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Notes

Note 1. Japanese references of MMT are Mochizuki (2020), Morinaga (2020), Nakano (2020), Park (2020), Shimakura (2019).

Appendix A

The consumption baskets and their prices are

$$C_1^e = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ C_2^e = \left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},$$

$$C_1^u = \left(\int_0^1 c_1^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ C_2^u = \left(\int_0^1 c_2^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},$$

$$P_1 = \left(\int_0^1 p_1(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}},$$

and

$$P_2 = \left(\int_0^1 p_2(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}}.$$

The budget constraint for an employed consumer is

$$\int_0^1 p_1(z)c_1^e(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2^e(z)dz + \frac{1}{1+r}B^e = Wl + \Pi + (1+r)\tilde{B}.$$

The budget constraint for an unemployed consumer is

$$\int_0^1 p_1(z)c_1^u(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2^u(z)dz + \frac{1}{1+r}B^u = \Pi + (1+r)\tilde{B}.$$

The Lagrange functions for them about consumptions are

$$\mathcal{L}^e = u\left(C_1^e, C_2^e, \frac{B^e}{P_2}\right) - \lambda\left(\int_0^q p_1(z)c_1^e(z)dz + \frac{1}{1+r}\int_0^q p_2(z)c_2^e(z)dz + \frac{1}{1+r}B^e - Wl - \Pi - (1+r)\tilde{B}\right),$$

and

$$\mathcal{L}^u = u\left(C_1^u, C_2^u, \frac{B^u}{P_2}\right) - \lambda\left(\int_0^q p_1(z)c_1^u(z)dz + \frac{1}{1+r}\int_0^q p_2(z)c_2^u(z)dz + \frac{1}{1+r}B^u - \Pi - (1+r)\tilde{B}\right).$$

We consider utility maximization for an employed consumer. The first order conditions about consumptions are

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_1^e}\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - \lambda p_1(z) = 0, \tag{A.1}$$

and

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_2^e}\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_2^e(z)^{-\frac{1}{\sigma}} - \lambda\frac{1}{1+r}p_2(z) = 0. \tag{A.2}$$

From them we obtain

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_1^e}\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz - \lambda\int_0^1 p_1(z)c_1^e(z)dz = 0,$$

and

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_2^e}\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} \int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz - \lambda\frac{1}{1+r}\int_0^1 p_2(z)c_2^e(z)dz = 0.$$

Then, we get

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_1^e}C_1^e - \lambda\int_0^1 p_1(z)c_1^e(z)dz = 0, \tag{A.3}$$

and

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial C_2^e}C_2^e - \lambda\frac{1}{1+r}\int_0^1 p_2(z)c_2^e(z)dz = 0. \tag{A.4}$$

On the other hand, from (A.1) and (A.2) we have

$$\left(\frac{\partial u\left(C_1^e, C_2^e, \frac{B^e}{P_2}\right)}{\partial C_1^e}\right)^{1-\sigma}\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1}\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz - \lambda^{1-\sigma}\int_0^1 p_1(z)^{1-\sigma}dz = 0,$$

and

$$\left(\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_2^e}\right)^{1-\sigma}\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1}\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz-\lambda^{1-\sigma}\left(\frac{1}{1+r}\right)^{1-\sigma}\int_0^1 p_2(z)^{1-\sigma}dz=0.$$

Thus, we obtain

$$\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_1^e}-\lambda\left(\int_0^1 p_1(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}}=0,$$

and

$$\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_2^e}-\lambda\frac{1}{1+r}\left(\int_0^1 p_2(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}}=0.$$

They are written as

$$\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_1^e}=\lambda P_1,\quad \frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial C_2^e}=\lambda\frac{1}{1+r}P_2. \tag{A.5}$$

Further, from (A.3) and (A.4) we get

$$P_1C_1^e=\int_0^1 p_1(z)c_1^e(z)dz,\quad P_2C_2^e=\int_0^1 p_2(z)c_2^e(z)dz.$$

By the budget constraint

$$P_1C_1^e+\frac{1}{1+r}P_2C_2^e+B^e=Wl+\Pi+(1+r)\tilde{B}. \tag{A.6}$$

The equations in (A.5) are the conditions for maximization of $u\left(C_1^e,C_2^e,\frac{B^e}{P_2}\right)$ about C_1^e and C_2^e subject to (A.6). The condition for maximization of $u\left(C_1^e,C_2^e,\frac{B^e}{P_2}\right)$ about the bequest is

$$\frac{1}{P_2}\frac{\partial u(C_1^e,C_2^e,\frac{B^e}{P_2})}{\partial B^e}=\lambda\frac{1}{1+r}.$$

This means

$$\frac{\partial u(C_1^e, C_2^e, \frac{B^e}{P_2})}{\partial B^e} = \lambda \frac{1}{1+r} P_2.$$

We can show similar results for an unemployed consumer. Since $u\left(C_1^e, C_2^e, \frac{B^e}{P_2}\right)$ and $u(C_1^u, C_2^u, \frac{B^u}{P_2})$ are homothetic,

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + \frac{1}{1+r} P_2 C_2^e + \frac{1}{1+r} B^e}, \ \beta = \frac{P_2 C_2^e}{P_1 C_1^e + \frac{1}{1+r} P_2 C_2^e + \frac{1}{1+r} B^e}$$

are determined by the prices, and do not depend on the income of consumers. Therefore,

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + \frac{1}{1+r} P_2 C_2^e + \frac{1}{1+r} B^e} = \frac{P_1 C_1^u}{(1+r) P_1 C_1^u + \frac{1}{1+r} P_2 C_2^u + \frac{1}{1+r} B^u},$$

$$\beta = \frac{\frac{1}{1+r} P_2 C_2^e}{P_1 C_1^e + \frac{1}{1+r} P_2 C_2^e + \frac{1}{1+r} B^e} = \frac{\frac{1}{1+r} P_2 C_2^u}{P_1 C_1^u + \frac{1}{1+r} P_2 C_2^u + \frac{1}{1+r} B^u}$$

hold. Also we have

$$1 - \alpha - \beta = \frac{\frac{1}{1+r} B^e}{P_1 C_1^e + \frac{1}{1+r} P_2 C_2^e + \frac{1}{1+r} B^e} = \frac{\frac{1}{1+r} B^u}{P_1 C_1^u + \frac{1}{1+r} P_2 C_2^u y + \frac{1}{1+r} B^u}$$

From these analyses we obtain the demand functions for consumption baskets as follows.

$$C_1^e = \frac{\alpha(Wl + \Pi + (1+r)\tilde{B})}{P_1}, \ C_2^e = \frac{\beta(Wl + \Pi + (1+r)\tilde{B})}{\frac{1}{1+r} P_2} \quad \text{(A.7)}$$

and

$$C_1^u = \frac{\alpha(\Pi + (1+r)\tilde{B})}{P_1}, \ C_2^u = \frac{\beta(\Pi + (1+r)\tilde{B})}{\frac{1}{1+r} P_2} \quad \text{(A.8)}$$

Also we have

$$\frac{B_2^e}{P_2} = (1 - \alpha - \beta) \frac{Wl + \Pi + (1+r)\tilde{B}}{\frac{1}{1+r} P_2}, \ \frac{B_2^u}{P_2} = (1 - \alpha - \beta) \frac{\Pi + (1+r)\tilde{B}}{\frac{1}{1+r} P_2},$$

or

$$B_2^e = (1-\alpha-\beta)(Wl + \Pi + (1+r)\tilde{B}), \ \ B_2^u = (1-\alpha-\beta)(\Pi + (1+r)\tilde{B}).$$

From (A.1), (A.2), (A.5)

$$P_1\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - p_1(z) = 0,$$

$$P_2\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_2^e(z)^{-\frac{1}{\sigma}} - \frac{1}{1+r}p_2(z) = 0.$$

This means

$$P_1^{-\sigma}\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-\frac{\sigma}{\sigma-1}} c_1^e(z) - p_1(z)^{-\sigma} = 0,$$

$$P_2^{-\sigma}\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-\frac{\sigma}{\sigma-1}} c_2^e(z) - \frac{1}{1+r}p_2(z)^{-\sigma} = 0.$$

Thus, we have

$$P_1^{-\sigma}\frac{1}{C_1^e}c_1^e(z) - p_1(z)^{-\sigma} = 0,$$

and

$$P_2^{-\sigma}\frac{1}{C_2^e}c_2^e(z) - \frac{1}{1+r}p_2(z)^{-\sigma} = 0.$$

From them and (A.7) we get the demand function for good z as follows.

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi+(1+r)\tilde{B})}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{\beta(Wl+\Pi+(1+r)\tilde{B})}{\frac{1}{1+r}P_2}.$$

Similarly, for an unemployed consumer by (A.8)

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(\Pi + (1+r)\tilde{B})}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{\beta(\Pi + (1+r)\tilde{B})}{\frac{1}{1+r}P_2}$$

are obtained. In the equilibrium all $p_1(z)$ are equal, and all $p_2(z)$ are equal.

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