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Is Walras's Theory So Different From Marshall's?

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Abstract

This paper shows that Marshall's theory is generally equivalent to Walras's one. It shows that Walras used two types of demand functions: (1) the original (ordinary) demand curve (function); and (2) the derived (general) demand function. Marshall also used both types of demand curves (function); however he did so in a very simplified and vague manner. Walras used a common method of equilibrium establishment and re-establishment of equilibrium for all four types of economies. First, he discussed the problem of equilibrium establishment using initial basic data. Second, Walras starts the adjustment process by using a model to describe the equilibrium state. He then describes the process of equilibrium establishment from a position of disequilibrium, by the use of his famous algorithm – *tâtonnement*. Finally, Walras discussed the problem of the variation of prices, or the problem of the re-establishment of equilibrium, as a result of changes in the initial basic data for any individual or any group of individuals. Marshall also used the same method, but in an incomplete form. Despite that, Marshall did not formulate the model for individual economies, he discussed the conditions of its optimality; and discussed the process of equilibrium establishment. Furthermore, despite the fact that Marshall described verbally *the complete model for the whole (macro) economy* in a similar way to Walras, he *did not formulate* mathematically that model. Marshall also discussed the problem of the re-establishment of equilibrium, as a result of changes in the initial basic data. Therefore, Friedman's statement that they are alternative theories is mistaken.

Keywords: Walras, Marshall, Friedman, Original and Derived Demand Curves (Functions)

1. Introduction

Over 60 years ago, in his seminal paper "The Marshallian Demand Curve", Friedman stated that there are two theories of demand: Marshall's and Walras's. Friedman also claimed that Marshall's interpretation of the demand curve is, in his view, wrong. He concluded his paper with the words: 'The current interpretation of the demand curve is Walrasian; and so is current economic theory in general' (Friedman, 1953, p. 93). Therefore, according to Friedman these two types of demand curves are different and mutually exclusive (Hirsch & de Marchi, 1990, p. 23; and pp. 33-34). Furthermore, Friedman stated that the Walras's demand function (curve) is the one which is generally used in modern economic theory.

Since that period in economic literature these two theories have been separated and named as either "the Walrasian demand curve (function)" or "the Marshallian demand curve (function)" without any explanation as to the differences between them. At the same time, there are economists who claim that there are differences between Walras's theory and Marshall's theory (De Vroey, 2009; Jaffe, 1971; Ingrao & Israel, 1990). The main difference is that Marshall's theory considers only partial equilibrium and not a general equilibrium theory, whereas Walras's theory considers only a general equilibrium theory (Screpanti & Zamangi, 1993, p. 178; Dasgupta, 1990, p. 245; Donzelli, 2008; Stigler, 1990, p. 5; Dardi, 2003; De Vroey, 1999a, 1999b; Hayes, 2006)).

Such differentiation between Walras's theory and Marshall's theory is not only incorrect, as will be shown in this paper, but also negatively reflects on the development of economic science; and it might cause serious harm when it is applied for practical recommendation (see recently published book of Cardenete et al., 2012, p. 101)*;* and there are some economists asserting that macroeconomics does not require micro foundations; and conversely, that microeconomics does not require macro connections.

Moreover, there are economists whom state that 'We have concluded that microeconomics does not provide knowledge that could not be obtained otherwise and that, as it is usually taught (or presented in textbooks), it encourages an erroneous way of thinking' and 'In a word, to understand the real world, one has to forget microeconomics' (Benicourt & Guerrien, p. 317 and p. 322 respectively).

Furthermore, the gloomy situation of the economic theory has had an influence on economic education, which itself deepens the crisis of economic theory, because that textbooks both micro and macroeconomics have been negatively influencing several generations of economic students.

At the same time, Negishi states that 'It cannot be denied, in any case, that Marshall's partial equilibrium analysis is an indispensable complement to Walras' general equilibrium analysis in forming the foundations of current mainstream economics' (Negishi, 1989, p. 345; see also Hutchison, 1953, p. 74).

It is necessary to stress that in the economic literature there are economists which notice the similarity of their approaches (Hicks, 1934, p. 338; Schumpeter, 1954, p. 837 and Whitaker, 1975, p. 103-104). On the other hand, there are authors who notice that Marshall's approach is more complex than Walras's one (Raffaelli, 2003, p. 91).

In this paper, however, it will be shown that Marshall's demand theory is essentially similar

to Walras's one, albeit that Walras's demand theory is comprehensive, whereas Marshall's demand theory is not. This paper concerns itself only with these issues which are significant both from the point view of the methodology of economic science and of course demand theory (Davar, 2014).

The paper consists of four sections. Following the introduction, in the first section, the first type of demand function (curve), namely, the original (ordinary) demand function is considered. In the second section, Walras's determination of the second type of demand function, namely, the derived (general) demand function is discussed; firstly in detail for the exchange economy, and then in brief, for other economies. The third section describes the link between these two types of demand functions. The fourth section considers the attributes of the derived demand function in Marshall's approach. Finally, conclusions are presented.

2. Original (Ordinary) Demand Curve (Function)

Let us start with Cournot's definition of the original (ordinary) demand curve (function): 'the sales or the demand generally, we say, increases when the price decreases' (Cournot, 1738, p. 46) and 'the sales or the demand D is, for each article, a particular function $F(p)$ of the price p of such article' (ibid. p. 47). This definition describes the character of the demand curve for a certain commodity and for any large market. It is clear, however that there might be some variations of the demand curve depending upon the character of the different commodities such as luxury and every-day consumed commodities. Therefore the demand curve for each individual differs and they may take on any form, may be discontinued and even sometimes inclined positively. However Marshall stated that 'There is then one general law of *demand*:- The greater the amount to be sold, the smaller must be the price at which it is offered in order to that it may find purchasers, or in other words, the amount demanded increases with a fall in price, and diminishes with a rise in price. There will not be any uniform relation between the fall in price and the increase of demand' (Marshall, 1952, pp. 98-99). Marshall also stated that 'Thus the one universal rule to which the demand curve conforms is that it is inclined negatively through the whole of its length' (ibid. p. 99, note 2; see also Marshall, 1930, p. 4).

On the other hand, Walras as well as Marshal stated that 'Thus, the slope of demand curve , which can be very simply defined in terms of mathematics as *the limit of the ratio of a decrease in demand to an increase in price*', and 'Hence the quantity demanded y is too great for a price higher than p_b. It follows therefore that '*the demand curve is negatively inclined*' (Walras, 1954, p.116 and p. 466, respectively). Walras assumed that demand and offer curves for an individual may be either continuous or discontinuous, whereas for total demand and total offer curves, they must be always continuous (ibid. p. 95).

Firstly, Walras determined the effective supply and effective demand as 'a definite amount of a commodity at a definite price' (ibid. p. 84 and p. 85). This means that for both demand and supply, for a particular quantity, there is only one price, and vice versa. Secondly, Walras determined the state of equilibrium by comparing the effective demand and the effective offer of a commodity: 'We have now to make three suppositions according as the demand is equal to, greater than, or less than the offer'. In the first case 'The market is in a stationary state or in equilibrium', while in the second and third cases 'the market is in disequilibrium' (Walras, 1954, p. 85).

On the one hand, Marshall determined what was *efficient demand* from the equilibrium state 'His demand becomes *efficient*, only when the price which he is willing to offer reaches that at which others are willing to sell' (Marshall, 1952, p. 95). Yet, Marshall used similar definition of equilibrium state but in inverse form, namely, the equilibrium is achieved when the demand price is equal to the supply price for the given quantity of commodity (vide infra).

There are two additional arguments showing the similarity of the demand theory of Walras and Marshall. Firstly, they both used a common method of establishment and re-establishment of general equilibrium, i.e., they assumed that the given basic data does not change during the process of equilibrium establishment and considered the problem of equilibrium re-establishment as a result of changes in the given basic data (Marshall, 1952, p. 342; Walras, 1954, p. 242). So, Marshall frequently used the term "other things are being equal" to point out the above statement. Walras stressed this fact when the process of equilibrium establishment first begins for each economy.

Secondly, both authors state that from the production economy demand curves of services and supply curves of commodities are not used and therefore, they are absent. However, in the process of equilibrium establishment the demand quantity of services is determined by the equation system which is based on the demand quantities of commodities. Furthermore the supply quantities of commodities are determined simultaneously with their demand quantities assuming that they are equal (vide infra).

To sum up we can conclude that Marshall's and Walras's definition of the original (ordinary) demand curve (function) for a certain commodity (service) are in principle similar (Schultz, 1938, p. 9).

3. Walras's Determination of the Derived (General) Demand Functions

It is well-known that Walras was the first author who used the demand (supply) function which is different from the original demand curve (function) where the quantity of a particular (certain) commodity depends not only on its price but also the prices of other commodities (Schultz, 1938, p. 9; Samuelson, 1947, p. 97). This fact was emphasised by most modern authors and referred to as a Walrasian demand function (see for example Mas-Colell & others, 1995). However, they modified Walras's derived demand function, and noted that the quantity of a certain commodity not only depends upon prices but also upon the other parameters of the initial endowment. In the following we will clarify this "difference" between Walras's and the modern authors' definition of the derived demand function.

Because in the majority of modern economic literature the problem of an individual economy is presented in relation to the exchange economy let us start our examination of the various economies from that point.

3.1 Walras's Derived (General) Function for the Exchange Economy

Walras stated that the following is the necessary and sufficient data in order to establish equilibrium in an exchange economy: '(1) the traders' utility or want equations for commodities, which can generally be represented by curves and (2) the initial quantities of the commodities in their possession' (Walras, 1954, p. 173). This means that each individual beforehand knows the holding quantities of commodities (q_1, q_2, ... , q_m), which he might

exchange with other individuals and thus formulates utility functions for every commodity separately ($\Phi_1(q_1)$, $\Phi_2(q_2)$, ... , $\Phi_m(q_m)$). It is necessary to point out that this data *does not change* during the whole process of equilibrium establishment. In these conditions, the goal of each individual is the maximum satisfaction of wants by the exchange of commodities for their given prices (p_1, p_2, ... , p_m), i.e., by the demand of one commodity and by the offer of others. In addition Walras assumed that every product might be either offered or demanded depending on its holding quantity. However, product cannot be offered if individuals do not hold it. Therefore Walras assumed that every commodity might be either demanded (d_i) or offered (o_i), such that the summation value of demand must be equal to the summation value of offer. This condition is known as *the budget constraint for individuals*.

Now, let us formulate a model in the exchange economy for each individual using the modern terms:

$$maximize \quad \sum_{i=1}^{m} \Phi_i(x_i), \tag{3.1}$$

subject to

$$x_i - d_i + o_i = q_i \,, \ (i = 2, 3, \ldots , m), \tag{3.2}$$

$$x_1 + \sum_{i=2}^{m} d_i p_i - \sum_{i=2}^{m} o_i p_i = q_1, \tag{3.3}$$

$$x_1, x_i, d_i \text{ and } o_i \geq 0 \,, \ (i = 2, 3, \ldots , m), \tag{3.4}$$

where

x_i - is the quantity of commodity i which remains to the individual by the result of exchange and it is calculated as either ($q_i + d_i$) or ($q_i - o_i$);

- conditions (3.2) indicates that the offer of a certain commodity cannot be more than its available quantity;

- condition (3.3) is the budget constraint for an individual, which means that either the offer of or demand for a commodity used as the *numéraire* (the first, according to Walras's approach) depends on the balance between the total value of demand (expenditure) $\sum_{i=2}^{m} d_i p_i$ and the total value of offer (income) $\sum_{i=2}^{m} o_i p_i$ of the commodities not used as the *numéraire.*

If $\sum_{i=2}^{m} d_i p_i > \sum_{i=2}^{m} o_i p_i$ then *the money commodity is offered in order to pay for the excess of the total value of demand* and it is defined as $o_1 = (\sum_{i=2}^{m} d_i p_i - \sum_{i=2}^{m} o_i p_i)$. Also the latter cannot be more than its available quantity q_1. And, if $\sum_{i=2}^{m} d_i p_i < \sum_{i=2}^{m} o_i p_i$ then *the money commodity is demanded in order to store (reserve) excess of the total value of supply* and it is defined as $d_1 = -(\sum_{i=2}^{m} d_i p_i - \sum_{i=2}^{m} o_i p_i)$.

The solution of the system (3.1)-(3.4), if it exists, determines the demand and offer quantities of commodities, which guarantees maximum satisfaction for each individual by the additional conditions '... that maximum satisfaction will be achieved by each trader when the ratios of

the *raretés* of the commodities not used as the *numerate* to the rareté of the commodity so used equal the prices cried' [Walras, 1954, p. 164]. This means that

$$\phi_i(x_i) = p_i\phi_1(x_1),\ (i=2,3,\ ...,m) \tag{3.5}$$

or

$$r_i = p_i\, r_1\ ,\ (i=2,3,\ ...,m) \tag{3.6}$$

where

$$r_i = \partial\Phi_i(x_i)/\ \partial x_i = \phi_i(x_i),\ (i=1,2,3,\ ...,m) \tag{3.7}$$

and r_i is known as the marginal utility (*rareté*) of commodity i for a certain individual.

From the solution to equations (3.1)-(3.4) (assuming it exists and is unique), we obtain demand quantities for a part of the commodities and offer quantities for the rest. And, consequently, the final quantity of good (x_i) is defined. Because of that the latter does not take place in the process of equilibrium establishment is omitted in the following discussion. It follows from the structure of the model that if either o_i or d_i is positive, the other would equal zero; because both have the same price and both influence the utility functions indirectly by the final endowment x_i (Hiller & Lieberman, 1995, pp. 586-591). In other words, a certain commodity cannot be offered and demanded simultaneously by the same individual. In addition, every commodity's offered quantities are bound by its available quantities (conditions (3.2)) and its demand quantities are bound by the available quantities of all commodities, i.e., by the budget constraints (condition (3.3)). This means that the derived (generally) demand (or offer) functions is a result of the solution of the model of individual economy, that is, it has pure theoretical nature. So:

$$o_i = f_i(p_1,p_2, p_3, ..., p_m; q_1, q_2, ..., q_m; \phi_1, \ldots, \phi_m);\ (i=2,3,\ ...,m) \tag{3.8}$$

$$d_i = f_i(p_1,p_2, p_3, ..., p_m; q_1, q_2, ..., q_m; \phi_1, \ldots, \phi_m\ );\ (i=2,3,\ ...,m) \tag{3.9}$$

while the individual's demand or offer of the *numéraire* (product (1)) is obtained by either the equation

$$o_1= (\sum_{i=2}^{m} d_i\, p_i - \sum_{i=2}^{m}\ o_i\, p_i) \text{ or } d_1= - (\sum_{i=2}^{m} d_i p_i - \sum_{i=2}^{m} o_i\, p_i), \tag{3.10}$$

where

ϕ_i – is the parameter of the utility function of product i.

Equilibrium conditions (3.5) or (3.6) are identical with Walras's one (ibid. p. 165). But, definitions of demand function (2.8) and offer function (3.9) differ from Walras's definition, namely from $y_1 = f_{b,1}(p_b, p_c, p_d\ ...)$, $z_1 = f_{c,1}(p_b, p_c, p_d\ ...)$, $w_1 = f_{d,1}(p_b, p_c, p_d\ ...)$' (Walras, 1954, p. 165). The functions (3.8) and (3.9) are determined on the basis of the solution of the mathematical model (3.1)-(3.4). Hence, by the theory of mathematical (linear) programming, unknowns of models depend upon all the parameters of the model. In this particular case they depend upon prices (p), available quantities (q), and the parameters of utility function of commodities' (ϕ). What this means is that Walras's demand and offer function is incomplete, i.e. is incorrect. However the question is why? Does Walras not know that such dependence is satisfied only by prices?

The parameters of Walras's individual model might be divided into two types: first, the internal parameters i.e. the initial available quantities and the utility functions of goods, and

secondly, the external - prices of commodities, which are generally unknowns. However, it must be pointed out, the prices become known for each iteration of adjustment process (*tâtonnement*) (vide infra). Consequently, Walras divided the process of equilibrium establishment into two stages. The first stage of the process is the establishment of equilibrium prices (external parameters) for the given available quantities and utility function (internal parameters). The second stage of the process is the analysis of the variation of prices (equilibrium re-establishment) when initial quantities and utility functions are changed. Thus, Walras's definition of derived (general) demand (supply) functions relates to the first stage. This means that the demand (or offer) of a certain commodity depends only on the prices of all commodities until general equilibrium is established. When equilibrium is established during the second stage of the process, then demand and supply is also dependant on the internal parameters (available quantity and utility of all goods) too. Walras used his famous *tâtonnement* for equilibrium establishment (vide infra).

It is necessary to stress that Walras's followers have misunderstood his two stage approach ever since Pareto (J. van Daal & D. A. Walker, 1990; Walker, 1996, 2006). They altered the first stage and though the second stage was used it was used in a different way from Walras's approach. For example, Samuelson writes: 'That is to say, the quantity of each good is a function of all prices and income. … These are the general demand functions. … The Marshallian partial equilibrium demand functions for the first good would be, of course, $x_1 = h^1(p_1, p_2, ..., p_n, I) = D^1(p_1)$, (21), where all other prices and income are held constant by *ceteris paribus* assumptions' (Samuelson, 1947, pp. 96-97). It is important to note two things. First, Samuelson considers only two parameters (prices and income) and the third parameter of utility functions is missing. Second, Samuelson assumes that the original demand function might be identified with the derived demand function and that it is acceptable for each individual. Samuelson's determination of the derived demand function was borrowed by the modern authors and called a "Walrasian demand function": 'When $x(p,w)$ is single-valued for all (p,w), we refer to it as the Walrasian (or *ordinary* or *market*) *demand function*' (Mas-Colell and others, 1995, p. 51)[1].

There is an additional erroneous argument made against Walras's approach. This argument is derived from a misunderstanding of Walras's two stage definition of the derived demand (offer) function. Walras's followers write as if Walras did not discuss problems of comparative static in his general equilibrium theory (Hicks, 1946, p. 61)[2].

Therefore, this means that in Walras's notations:

$$o_i = f_i(p_2, p_3, ..., p_m); (i=2,3, ...,m) \quad (3.11)$$

$$d_i = f_i(p_2, p_3, ..., p_m); (i=2,3, ...,m) \quad (3.12)$$

while the individual's demand or offer of the *numéraire* product (1) is obtained by the equation (2.10).

3.2 Derived (General) Function for Another Three Economies

3.2.1 Derived Function for Production Economy

In the production economy Walras considered two distinct markets. The first is the services market, where owners of factors either sell various productive services, which are bought by entrepreneurs or other individuals for productive purposes or the services are bought by other

individuals for the purposes of consumption. The second market is the products market, where entrepreneurs sell their products and individuals buy them for the purposes of consumption. Here, as well as, in an exchange economy, productive services and products are exchanged by the rule of free competition. In addition, their prices (rent, wages, interest, and prices, respectively) are stated in terms of *numéraire.* Moreover, the current (equilibrium) price of each service or each product is established in accordance with the law of supply and demand. In other words, by the relationship between effective demand and effective offered (between selling prices (demand prices) of commodities and their cost of production (supply prices)).

Each individual, at this stage, exchanges his own capital services for the products and other individual services. Therefore, in addition to the previous Exchange Economy, where utility functions of each product are given, here each individual has to define utility functions for every service separately ($\Phi_j(y_j)$) and their available quantity. It is necessary to point out that in the production economy Walras assumed that individuals do not possess any quantities of products, and therefore, available quantities of products from the previous economy are absent; and 'in a state of equilibrium in production, entrepreneurs make neither profit nor loss' (Walras, 1954, p. 225). In this section we did not differ between types of services and assumed that their total number is equal to n.

Therefore the derived demand function for the production economy is an extended version of the Exchange Economy (see (3.8) and (3.9)). The complete derived demand (supply) function of products (services) obtained on the basis of individual models' solution for the production economy must include prices of products and services, the available quantities of services and the parameters of utility functions of products and services must be included. Thus:

$$d_j = f_j(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n; q_1, q_2, ..., q_n; \phi_1, \ldots, \phi_m; \phi_1, \ldots, \phi_n); \quad (j=1,2,3, ...,n) \tag{3.19}$$

$$o_j = f_j(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n; q_1, q_2, ..., q_n; \phi_1, \ldots, \phi_m; \phi_1, \ldots, \phi_n); \quad (j=1,2,3, ...,n) \tag{3.20}$$

$$x_i = f_i(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n; q_1, q_2, ..., q_n; \phi_1, \ldots, \phi_m; \phi_1, \ldots, \phi_n); \quad (i=2,3, ...,m) \tag{3.21}$$

while the individual's demand of the *numéraire* (product (1)) is obtained by the equation

$$x_1 = -\sum_{i=2}^{m} x_i p_i - \sum_{j=1}^{n} d_j p_j + \sum_{j=1}^{n} o_j p_j, \tag{3.22}$$

Where x_i, d_j, and o_j are the demand for products and services, and the offer of services respectively.

Due to this Walras's version of the derived demand function for production economy has to be also an extended version of the exchange economy (see (3.10) and (3.11). This means that

$$d_j = f_j(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n;); \ (j=1,2,3, ...,n) \tag{3.23}$$

$$o_j = f_j(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n;); \ (j=1,2,3, ...,n) \tag{3.24}$$

$$x_i = f_i(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n;); \ (i=2,3, ...,m) \tag{3.25}$$

$$x_1 = -\sum_{i=2}^{m} x_i p_i - \sum_{j=1}^{n} d_j p_j + \sum_{j=1}^{n} o_j p_j, \tag{3.26}$$

However, in the equilibrium state, for the given available quantities and utility function ϳn the first stage of the equilibrium establishment process, there will always be agents (consumers and producers) which stay outside of the equilibrium state. The result is that they can change their given available quantity and utility function and therefore, the second stage of the equilibrium establishment process is required.

3.2.2 Derived Function for Capital Formation and Credit Economy

Capital formation and credit economy are extended by the production of new capital goods, which are required for two purposes. The first is the renewal of the old capital goods, which have been destroyed, in order to keep up the existing level of total production if it is required. The second purpose is to expand existing fixed capital. On the other hand, in order to demand (purchase) new capital goods there must be individuals whose incomes exceed their purchase of consumers' goods and services, so that the aggregate of the first is greater than the aggregate of the latter, i.e. there is a saving.

In order to convert this new term, a saving, to a term which would be comprehensive, that is, one which would be similar to other consumers goods, Walras introduced an abstract (ideal) commodity (E) consisting of *perpetual net income* with price $p_e = 1/i$. This means that each individual has a certain want of commodity (E) which is either demanded (d_e) or offered (o_e) as well as other capital services and quantity of which is obtained by the condition of maximum satisfaction by its function of utility $\Phi_e(q_e)$.

So, the derived demand function for the Capital Formation and Credit Economy is an extended version of the Production Economy (see (23.19) - (3.21)). The complete derived demand (supply) function of products (services) obtained on the basis of an individual model for Capital Formation and Credit Economy must include the following additional parameters relating to the derived demand function of a Production Economy: price (p_e) of a new commodity (E), its available quantity (q_e), and the parameter of its utility function (ϕ_e). This is:

$$d = f(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n; p_{k'}, p_{k''} \ldots; p_e; q_1, q_2, ..., q_n; q_e: \phi_1, \ldots, \phi_m; \phi_1, \ldots, \phi_n; \phi_e); \quad (3.26)$$

Where, for simplicities sake we use d to express a common notation for demand and supply functions for all kinds of commodities and services.

Walras's version of a derived demand function for Capital Formation and Credit Economy then has to be also an extended version of the Production economy (see (3.23) - (3.25)). This means that according to Walras's approach the derived demand function has the following form

$$d = f(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n; p_{k'}, p_{k''} \ldots; p_e); \quad (3.27)$$

In the capital formation and credit economy alike the exchange economy and production economy, in the equilibrium state, for the given available quantities and utility function ϳn the first stage of the equilibrium establishment process, there will always be agents (consumers and producers) which stay outside of the equilibrium state. The result is that they can change their given available quantity and utility function and therefore, the second stage of the equilibrium establishment process is required.

3.2.3 Derived Function for a Circulation and Money Economy

Circulation and Money is Walras's final study of economy. In this economy Walras extended

three previous economies (Exchange, Production, and Capital Formation and Credit economies) by introducing the circulation of capital goods, money, and raw materials.

So, the derived demand function for the Circulation and Money Economy is an extended version of the Capital Formation and Credit Economy (see (3.26) and (3.7)). The complete derived demand (supply) function of products (services) obtained on the basis of an individual model for the Circulation and Money Economy must include a number of additional parameters relating to the derived demand function of Capital Formation and Credit. The following are the additional parameters : the price ($p_{u'}$) of a circulating money's service, its available quantity (q_u); prices of the services of availability of products as circulating capital goods ($p_{a'}$) and their available quantity ($q_{a'}$) the parameter of its utility function ($\phi_{a'}$); and raw materials' prices ($p_{r'}$) in circulation and their available quantities of (q_m); thus:

$$d = f(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n; p_{k'}, p_{k''} \ldots; p_{1'}, p_{2'}, ..., p_{m'}; p_{r'}; p_{u'}; p_e; q_1, q_2, ..., q_n; q_{1'}, q_{2'}, ..., q_{m'}; \quad q_1, q_2, ..., q_r; q_u; \quad q_e: \phi_1, \ldots, \phi_m; \phi_1, \ldots, \phi_n; \phi_{1'}, \ldots, \phi_{m'}; \phi_e); \tag{3.28}$$

Where, as in the previous economy, for simplicities sake we use, d to express a common notation for both demand and supply functions for all kinds of commodities and services.

Walras's version of a derived demand function for Circulation and Money Economy then has to be also an extended version of the Capital Formation and Credit Economy (see (3.27)). This means that according to Walras's approach the derived demand function has the following form

$$d = f(p_2, p_3, ..., p_m; p_1, p_2, ..., p_n; p_{k'}, p_{k''} \ldots; p_{1'}, p_{2'}, ..., p_{m'}; p_{r'}; p_{u'}; p_e); \tag{3.29}$$

4. The Link between These Two Types of Demand Functions

There are two types of demand (supply) functions: the original (ordinary) demand (supply) curve (function) and the derived (general) demand (supply) functions for commodities and factors. The original demand function means that the quantity of the commodity in question depends only on its price and vice versa, the price of the commodity in question depends only on its quantity. This means that the original demand function is an invertible function and it might be drawn, i.e., there is original demand curve. The derived (general) demand function means that the quantity of the commodity in question depends on the prices of all commodities and services including money. The derived demand function is not invertible opposed to the original demand function[3]. The problem which we must now turn our attention to is what kind of relationship is between these functions, i.e., whether they are alternative, substitutable or whether they can co-exist and be used simultaneously. In order to answer this question it is necessary to consider the process of general equilibrium establishment for the exchange economy in detail.

Walras stated that in order to any random prices to become equilibrium prices it is necessary for the total effective demand to equal the total effective supply for all commodities which obtained by the aggregation of the results of the solution of individuals' economy. Therefore, Walras formulated two equivalent types of equation systems describing the equilibrium state. First, the excess demand (offer) for all commodities must equal zero. Second, the total demand must equal the total offer for all commodities except the commodity used as *numéraire*, where equilibrium would be consequently established.

Based upon one of the essential assumptions that total effective demand must equal total effective supply, Walras (Walras, 1954, p. 168) formulated equation systems for the whole economy for the determination of current (equilibrium) prices, thus:

$$D_i(p_2, ..., p_m) = O_i(p_2, ..., p_m), (i=2,3, ...,m) \quad (4.1)$$

Walras also stated that since prices of commodities 'are by their nature positive, it is evident that, if the above equations are satisfied ... we also have' (Ibid. p.169).

$$D_1 - O_1 = \sum_{i=2}^{m} D_i p_i - \sum_{i=2}^{m} O_i p_i = 0, \quad (4.2)$$

It is necessary to point out that these systems of equations describe an equilibrium situation for the whole economy in the Exchange Economy. But the question is how Walras achieved this equilibrium situation. Generally, there might be three situations for each commodity: 1) where its total demand is greater than its total offer; 2) where its total offer is greater than its total demand; and 3) where its total demand equals its total offer:

$$D_i(p_2', ..., p_m') >=< O_i(p_2', ..., p_m') , (i=2,3, ...,m) \quad (4.3)$$

This means that there is a disequilibrium state, and in such situation, Walras used a characteristic of the original demand function in order to carry out the process of general equilibrium establishment. On the basis of these demand curves Walras stated that for the first situation, the price of the commodity will increase in order to decrease demand quantity. In the second situation the price will decrease in order to increase the demand quantity. Finally in the third situation the price will not change. Walras used these rules of change in prices in the real market in his theoretical solution for the establishment of equilibrium (Walras, 1954, p. 170). This means that in order to establish an equilibrium situation Walras used *tâtonnement* (iterative process), where the iteration consists of several stages of comparison between the total demand and the total offer for each commodity of which inequality exists. So that at each stage first a new price system is determined on the basis of the original demand curve according to whether there is an equilibrium or disequilibrium situation for each commodity. Then using this given price system each individual determines his own derived demand (offer) functions and by means of these individual demand and offer quantities, the total demand and the total offer are determined in order to establish whether there is equilibrium. In other words it is necessary to compile data to determine a new price system for the next stage and to determine whether it would be necessary. The number of stages is equal to the number of commodities for which inequality exists. So, Walras used here the property of the original demand function. According to the rules of the real market, he stated that, it is possible *to establish partial equilibrium*, i.e. equilibrium for a certain commodity if it exists at all (problems of existence of equilibrium will not be discussed here). Walras stated, therefore, that by continuing the same way for the new price system, the process moves closer and closer to a state of equilibrium. Walras formulated the *law* of the establishment of equilibrium prices for the exchange economy for the given data (ibid):

Given several commodities, which are exchanged for one another through the medium of a numéraire, *for the market to be in a state of equilibrium or for the price of each and every commodity in terms of the* numéraire *to be stationary, it is necessary and sufficient that at these prices the effective demand for each commodity equal its effective offer. When this*

equality absent, the attainment of equilibrium prices requires a rise in the prices of those commodities the effective demand for which is greater than effective offer, and fall in the prices of those commodities the effective offer of which is greater than the effective demand.

This law allows us to conclude that Walras's original *tâtonnement,* the iterative process of equilibrium establishment, is a *theoretical version* of the equilibrium establishment process in a real market. In addition, in this law Walras expressed directly that the quantity of demand of a certain commodity is changed according to the original demand function, namely in relation to its price only. This means that in the process of equilibrium establishment Walras used both demand functions: original (ordinary) and derived (general). It is necessary to stress that Walras's original law differs from "Walras' Law" which is used in modern economic literature (Davar, 2012).

After equilibrium establishment Walras discussed problems of the variation of prices when the given data, utility function and initial endowment of commodities, is changed for some individual or a group of individuals. Without discussing Walras's original presentation of the problem it is still important to stress that Walras formulated the Law of Variation of Commodity prices in an Exchange Economy (Walras, 1954, p. 180).

5. Attributes of the Derived Demand Functions in Marshall's Demand Theory

Marshall did not define, unfortunately, the derived demand function obviously. However a studied look of the text and mathematical appendix of his *Principles of Economics*, leads us to conclude that Marshall also used this function in his theory. The main reason that Marshall did not clearly consider the derived demand function, in our opinion, is that *not only did he not formulate any mathematical model for an individual economy* **(**Dardi, 2003, p. 1) *but also he did not formulate a complete model for the whole* (*macro*) *economy.* Therefore, he could not discuss the adjustment process between an individual economy and a macro economy in the same way that Walras did. This is also the reason, by our opinion, why majority of economists call Marshall's theory a partial equilibrium theory, and not a general equilibrium theory. However, Marshall determined the condition of optimality for a model of an individual's economy and formulated fragments of a whole economy in the same way Walras did. This shows that he generally discussed a General Equilibrium Theory, as did Walras (vide infra).

There are many attributes, but we will confine ourselves only to those which are relevant and central to the subject under discussion.

5.1 Attribute for the Individual Economy in Marshall's Demand Theory

Despite the fact that Marshall did not formulate and discuss a mathematical model for an individual economy, he determined exactly the condition of optimality for an individual economy. Marshall firstly formulated a general rule, stating that 'If a person has a thing which he can put to several uses, he will distribute it among these uses in such a way that it has the same marginal utility in all. For if it had a greater marginal utility in one use than another, he would gain by taking away some of it from the second use and applying it to the first' (Marshall 1952 pp. 117-118). Then he concretized this rule for money stating that

But when commodities have become very numerous and highly specialized, there is an urgent need for the free use of money, or general purchasing power; for that alone can be applied easily in an unlimited variety of purchases. And in a money-economy, good management is shown by so adjusting the margins of suspense on each line of expenditure that the marginal utility of a shilling's worth of goods on each line shall be the same (ibid. p.118).

It is worthy noting two points. Firstly, already at the level of the individual economy Marshall considered numerous numbers of commodities. Secondly, he discussed an amount of money but not money income (Samuelson, 1947, p. 100 and p.190, see note 6). Marshall stated that 'The larger the amount of a thing that a person has the less, other things being equal (i.e. the purchasing power of money, and *the amount of money at his command* (our italics) being equal), will the price which he will pay for a little more of it: or in other words his marginal demand price for it diminishes' (Marshall, 1952, p. 95).

Regarding money he concluded, that 'At one and the same time, a person's material resources being unchanged, the marginal utility of money to him is a fixed quantity, so that the prices he is just willing to pay for two commodities are to one another in the same ratio as the utility of those two commodities' (ibid. p. 95). This means that the marginal utility of money stood fixed until the initial given parameters for individuals do not change and it is the same to relations for all commodities. This is identical with Walras's approach.

Marshall considered a mathematical expression of this condition of optimality for the individual economy in Note II in his mathematical appendix (ibid. p. 838). He wrote:

If m is the amount of money or general purchasing power at a person's disposal at any time, and μ represents its total utility to him, then $d\mu/dm$ represents the marginal degree of utility of money to him.

If p is the price which he is just willing to pay for an amount x of the commodity which gives him a total pleasure u, then

$$d\mu/dm \cdot \Delta p = \Delta u; \text{ and } d\mu/dm \cdot dp/dx = du/dx.$$

If we take into account the fact that by p Marshall notated the price paid for an amount of commodity, and therefore dp/dx is the price of the unit commodity, we can conclude that this last expression is identical with Walras's conditions of optimality for an individual economy (see (3.5)). Namely, the maximum satisfaction for a person will be achieved when the ratios of the marginal utility of any commodity (which is not used as the *numéraire* (money commodity)) to the marginal utility of the money commodity is equal to the price of commodity in question[4].

Marshal continued (ibid.):

If p' is the price which he is just willing to pay for an amount x' of another commodity, which affords him a total pleasure u', then

$$d\mu/dm \cdot dp'/dx' = du'/dx'$$

and therefore

$$dp/dx : dp'/dx' = du/dx : du'/dx'.$$

The latter expression is an extended version of the above conditions of optimality for an individual economy, which means that the ratio of the marginal utility of two commodities is equal to the ratio of their prices. This statement is identical to Walras's according statement (Walras, 1954, pp. 472-473).

To sum up, we can conclude that the conditions of optimality for the individual economy according to Marshall's approach are the same conditions that Walras set.

5.2 Attributes for the Whole Economy in Marshall's Theory

In this section, despite the fact that Marshall did not formulate a complete model of the whole economy and did not discuss the process of adjustment (equilibrium establishment) between the individuals' economy and the whole economy, we will show that generally Marshall's approach is equivalent to Walras's one.

Firstly, Marshall as well as Walras assumed that there would be free competition and uniform prices. He wrote: 'Thus we assume that the forces of demand and supply have free play; that there is no close combination among dealers on either side, but each acts for himself, and there is much free competition; that is, buyers generally compete freely with buyers, and sellers compete freely with sellers' (Marshall, 1952, p. 341; and 1930, p. 1); and 'we assume that there is only one price in the market at one and the same time' (Marshall, 1952, pp. 341-342).

Secondly, Marshall stressed repeatedly that he formulated mathematically only fragments of the whole model despite the fact that he depicted theoretically (verbally) the whole macro model. For example, Marshall, firstly, stated that 'In the theory of Domestic values it is not necessary to consider at one time the special circumstances of more than one commodity;' (Marshall, 1930, p.2), and then he discussed how equilibrium is established.

Marshall started with the following definition: 'Definition. *R* (Fig. 22A) being a point on *Ox*, let *OR* measure the amount of commodity which would be produced in a year if the scale on which the production is carried on at a given time were continued uniformity. Then *R* is the *Amount*-index at that time' (Marshall, 1930, p. 10).

This definition is similar to Walras's determination of effective demand and effective supply, but in inverse form. Namely, according to Walras's approach, the effective demand and the effective supply are determined for a certain given price; while according to Marshall's approach the Amount-index determines the demand price and the supply price for a certain given quantity.

Marshall continued: 'With this definition we may enunciate the fundamental

Prop. XIX. *Let a vertical straight line drawn through the Amount-index cut the demand curve in* d, *and the Supply curve in* s. *If* d *is above* s *the Amount-index will tend to move to the right. If* d *is below* s *the Amount-index tend to move to the left. If* d *coincides with* s, *the Amount-index will be in equilibrium, tending to move neither to the right nor to the left*' (ibid.).

This is also similar to Walras's rule of the establishment of equilibrium, but also in inverse

form. Namely, according to Walras's approach, the price of the commodity, in a disequilibrium situation, changes depending on whether there is either excess demand or excess supply; in the first situation the price must increase; in the latter situation, the price must decrease (vide supra). Thus, according to Marshall's approach, the quantity of the commodity, in a disequilibrium situation, changes depending on either if the price of demand is greater than the price of supply (the excess demand price), or the price of supply is greater than the price of demand (the excess supply price). In the first case the quantity must increase, while in the latter, it must decrease. It must pointed out, however, that Marshall did not use the terms "excess demand price" and "excess supply price". Marshal illustrated his approach for a local corn market (see Marshall, 1952, pp. 332-335).

Marshall finished by the defining the equilibrium state: 'Prop. XX. *The Amount-index is in equilibrium whenever it is vertically below any point of intersection of the Demand and Supply curves*' (Marshall, 1930, p.11). In other words, the equilibrium is established when the demand price is equal to the supply price for the given quantity of commodity; which is also similar to Walras's determination of the equilibrium state, but in inverse form, namely, the equilibrium is established when the effective demand is equal to the effective supply for the given price of commodity.

It must be stressed that when several commodities are discussed, which must be generally considered, the use of Marshall's method is problematic, since in this case, the aggregate demand and supply are simultaneously defined for several commodities, and this is only possible when their prices are given, i.e., by Walras's method.

Now let us to discuss Marshall's version of the theoretical (verbal) macro model of the whole economy. He wrote Marshall (1952, Note XXI, p. 855):

We may now take a bird's-eye view of the problems of joint demand, composite demand, joint supply and composite supply when they are all arise together, with the object of making sure that our abstract theory has just as many equations as it has unknowns, neither more nor less.

In a problem of joint demand we may suppose that there are n commodities $A_1, A_2, \ldots A_n$. Let A_1 have a_1 factors of production, let A_2 have a_2 factors, and so on, so that the total number of factors of production is $a_1+a_2+a_3+\ldots+a_n$; let this $=m$.

First, suppose that all the factors are different, so that there is no composite demand; that each factor has a separate process of production, so that there are no joint products; and lastly, that no two factors subserve the same use, so that there is no composite supply. We then have $2n+2m$ unknowns, viz. the amounts and prices of n commodities and of m factors; and to determine them we have $2n+2m$ equations, viz.-(i) n demand equations, each of which connects the price and amount of a commodity; (ii) n equations, each of which equates the supply price for any amount of a commodity to the sum of the prices of corresponding amounts of its factors; (iii) m supply equations, each of which connects the price of a factor with its amount; and lastly, (iv) m equations, each of which states the amount of a factor is used in the production of a given amount of the commodity.

Marshall described here a general equilibrium model for the Production economy which is similar to Walras's one, with minor differences (Walras, 1954, Lesson 20, pp. 237-242; see also Davar, 1994, pp.38-52). First of all, there is the equilibrium state, which is established by comparing between: (1) the demand price of a commodity (equations (i)) and its supply price (cost of production) (equations (ii)); and between (2) the supply quantities of factors (equations (iii)) and their demand quantities (equations (iv)). Secondly, Marshall, as well as Walras, used only the supply functions of the factors' services and the demand functions for commodities from the Production Economy in order to establish equilibrium. Thirdly, if due to the change in prices and quantities in any of the above comparisons there is inequality and therefore disequilibrium the process of adjustment must be continued until equilibrium is established if it exists at all.

Let us present Marshall's illustration of determination both of the cost of production and the derived demand quantities of factors. For his determination of the cost of production Marshall stated that (Marshall, 1952, p. 343):

'Let us suppose that a person well acquainted with the woollen trade sets himself to inquire what would be the normal supply price of a certain number of millions of yards annually of a particular of cloth. He would have to reckon (i) the price of the wool, coal, and other materials which would be used up in making it, (ii) wear-tear and depreciation of the buildings, machinery and other fixed capital, (iii) interest and insurance on all the capital, (iv) the wages of those who work in the factories, and (v) the gross earnings of management (including insurance against loss), of those who undertake the risks, who engineer and superintend the working.

This is a very comprehensive determination of the cost of production for the woollen industry and is similar to Walras's one.

For the latter, namely for the derived demand quantities of services Marshall stated that

The demand for raw materials and other means of production is *indirect* and is *derived* from the direct demand for those directly serviceable products which they help to produce. … there is a *joint demand* for the services which any of these things render to helping to produce a thing which satisfies wants directly and for which there is therefore a direct demand: the direct demand for the finished product is in effect spilt up into many derived demands for the things used in producing it (Marshall, 1952, p. 381).

This is also similar to Walras's definition of the demand of the factors' services (vide supra).

Marshall continued that 'When therefore the amount produced (in a unit time) is such that the demand price is greater than the supply price, then sellers receive more than is sufficient to make it worth their while to brings goods to market to that amount; and there is at work an active force tending to increase the amount brought forward to sale. On the other hand, when the produced is such that the demand price is less than the supply price, sellers receive less than is sufficient to make worth their while to bring goods to market on that scale; so that those who were just on the margin of doubt as to whether to go on producing are decided not to do so, and there is an active force at work tending to diminish the amount brought forward

to sale. When demand price is equal to the supply price, the amount produced has no tendency either to be increased or to be diminished; it is in equilibrium' (ibid. p. 345).

So, Marshall, as well as Walras, considered equilibrium establishment as taking place simultaneously in two directions. First, when the demand price of a certain commodity is greater than its supply price (cost of production) then the amount of commodity produced tends to increase due to increased quantities of factors with accordingly their high prices and consequently enlarging supply prices. Furthermore in the opposite case, namely, when the demand prices of a certain commodity is less than its supply price then the amount of the commodity produced tend to decrease by using decreased quantities of factors with accordingly their lower prices and consequently their decreasing supply prices. In this case *the adjustment process is carried out on basis of the original supply curves of factors.* Second, when the supply quantity of a certain factor is greater than its derived demand quantity, then the quantity of a commodity demanded tends to increase. This consequently increases the used quantities of factors. Thus when the supply quantity of factor is less than their required quantity then the quantity of commodity demanded tends to decrease and decreases the used quantities of factors. Therefore in this case *the adjustment process is carried out on the basis of the original demand curves of commodities*. In other words, Marshall also used identically expressed Walras's excess demand (supply) for the same purpose, but he did not define excess demand (supply) (see Marshall, 1952, note 1, p. 346).

Finally, Marshall, as well as Walras, stated that if there is any change in the initial given data for any individual of group of individuals then the new process of equilibrium establishment must be carried out. This means that equilibrium must be re-established. Marshal stated that 'For indeed the demand and supply schedules do not in practice remain unchanged for a long time together, but are constantly being changed; and every change in them alters equilibrium amount and equilibrium price, and thus gives new position to the centres about which the amount and the price tend to oscillate' (Marshall, 1952, pp. 346-347).

At the same time, there are economists which claiming that as if there is an important difference between Walras's and Marshall's determination of cost of production. Namely, they asserted that Walras used "coefficients of production" (Walras, 1954, p. 239), this is, average technology, while Marshall used a "representative firm". Indeed, Marshall stated that 'These results will be of great importance when we come to discuss the causes which govern the supply price of a commodity. We shall have to analyse carefully the normal cost of producing a commodity, relatively to a given aggregate volume of production; and for this purpose we shall have to study *the expenses of a representative producer* for that aggregate volume' (Marshall, 1952, p. 317).

But question is how Marshall determined a representative firm. Marshall stated that 'Thus a representative firm is in a sense an average firm. But there are many ways in which the term "average" migth be interpreted in connection with a business. And a Representative firm is that particular sort of average firm, at which we need to look in order to see how far the economies, *internal and external*, of production on a large scale have extended generally in the industry and country in question' (ibid. p. 318). So, Marshall also used average

technology as well as Walras, and the difference might be in the method of the calculation of the average technology.

To sum up, despite the fact that Marshall did not formulate the complete mathematical model of a whole economy and did not discuss the process of equilibrium establishment in detail, we can conclude that the method of equilibrium establishment of the whole economy according to Marshall's approach is equivalent to Walras, with some insignificant differences.

6. Conclusions

This paper shows that despite the fact that there are some differences between Walras's and Marshall's demand theory, *their approaches are essentially equivalent*.

Walras in his approach used two types of demand functions and in the process of equilibrium establishment using them, made connections between them. Marshall also used both types of demand curves; however he did so in a very implicit, simplified and vague manner and did not discuss any connection between them.

(1) Both, Walras and Marshall, from the outset defined the demand function as being where the demand quantity of a certain commodity depends only on its price and conversely the price of any commodity depends only on its quantity. In other words, there is a reciprocal connection between quantity and price for a particular commodity. It follows therefore that these functions might be invertible functions. Let us call this demand function the *original (ordinary) demand function (curve)* – $(d_i = f_i(p_i))$.

(2) Walras also used the second type of demand function where the demand quantity of any commodity depends on the prices of all commodities. Here the principle of mutuality and therefore inevitability does not hold. This is because it is impossible to say that the price of any good depends on the quantities of all commodities. One of the reasons for this is that in this case the demand function is obtained by the solution of the model for each individual for any system of random prices. Let us call this demand function the *derived (general) demand function* – for an exchange economy - $(d_i = f_i(p_2, p_3, ..., p_m))$. It must be stressed that this demand function varies depending on the type of economy in question. In every subsequent economy a number of prices are extended depending on the new commodities added to this economy. For example, in the production economy the prices of services are added - $(d_i = f_i(p_2, p_3, ..., p_m; q_1, q_2, ..., q_m))$; etc.

Marshall, unfortunately, did not clearly formulate the derived demand function, but examining his descriptions of some aspects of demand theory it might be understood that they are based on the secondary demand function. The main reason that Marshall did not clearly formulate the secondary demand function is that *he never formulated any mathematical model for an individual economy*. However, Marshall determined the condition of optimality for a model of an individual economy in the same way Walras did.

(3) Walras first defined *effective demand* and *effective supply*, and then used them as conditions for an equilibrium state. Marshall on the other hand, first studied the equilibrium state from which he then derived his definition of *efficient* demand.

(4) In consequence, Walras defined excess demand as the relationship between effective demand and effective supply, and used it in the process of equilibrium establishment. While,

Marshall used the relationship between demand price and supply price in the process of equilibrium establishment but he did not used the terms "excess demand price" and "excess supply price".

(5) Walras used a common method of equilibrium establishment and re-establishment of equilibrium for all four types of economies (Davar, 2014). First, he discussed the problem of equilibrium establishment using initial basic data. The total demand and the total offer of goods and services are obtained by aggregating the results from the solution of individual models. Second, Walras starts the adjustment process by using a model (simultaneous equation system) to describe the equilibrium state. He then describes the process of equilibrium establishment from a position of disequilibrium, which is described using equation systems where the number of unknowns is larger than the number of equations. This was done by the use of his famous algorithm – *tâtonnement*. Finally, Walras discussed the problem of the variation of prices, or the problem of the re-establishment of equilibrium, as a result of changes in the initial basic data for any individual or any group of individuals.

Marshall also used the same method, but in an incomplete form. Despite that, Marshall did not formulate the model for individual economies, he discussed the conditions of its optimality; and discussed the process of equilibrium establishment, albeit only for a single (particular) commodity. Furthermore, despite the fact that Marshall described verbally *the complete model for the whole (macro) economy* in a similar way to Walras, he *did not formulate* mathematically that model. Therefore, he could not discuss the adjustment process between an individual economy and a macro economy in the same way that Walras could. Marshall also discussed the problem of the re-establishment of equilibrium, as a result of changes in the initial basic data.

(6) Walras, in the adjustment process, connected between two types of demand functions. On the basis of the solution of individual models the derived demand functions define the aggregate demand function together with the aggregate supply function, in order to determine whether there is equilibrium. If there is disequilibrium then the original demand function is used in order to determine the required changes of prices or quantities for the next iteration of adjustment if it is needed. So, in Walras' approach the original demand function is only used on the macro level, while the derived demand function is used on both levels. *In the adjustment process they are interconnected*.

To summarise, despite the fact that there are some differences between Walras's and Marshall's demand theory, *their approaches are essentially equivalent*. So, such differentiation between Walras's theory and Marshall's theory is not only incorrect, but also negatively reflects on the development of economic science; and it might cause serious harm when it is applied for practical recommendation

Therefore, Friedman's statement that they are alternative theories *is mistaken*. This mistake of Friedman is based upon his misunderstanding and misinterpretation of both Walras's and Marshall's demand theories.

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Endnotes

[1] The authors note: 'This demand function has also been called the *Marshallian demand function*. However, this terminology can create confusion, and so we do not use it here. In Marshallian partial equilibrium analysis (where wealth effects are absent), all the different kinds of demand functions studied in this chapter coincide, and so it is not clear which of these demand functions would deserve the Marshall name in the more general setting' (Mas-Colell and others, 1995, p. 51).

[2] 'It was an achievement of the first magnitude for the older mathematical economists to have shown that the number of independent and consistent economic relations was in a wide variety of case sufficient to determine the equilibrium values of unknown economic prices and quantities. Since their life spans were only of finite duration, it was natural that they should have stopped short at the stage of counting equations and unknowns. … It is the task of comparative statics to show the determination of the equilibrium values of given variables (unknowns) under postulated conditions (functional relationships) with various data (parameters) being specified' (Samuelson, 1947, p. 257).

[3] 'They also often "inverted" the demnad function and wrote

$$p_i = f_i(x_1, \ldots, x_n)$$

This simplifies the mathematics, but is quite illegitimate. Entirely apart from the classical indeterminacy of absolute prices (which can easily be allowed for in Wald's formulation and which actually was taken explicitly into account by him) this version says that any configuration of market demands can be brought about by one and only one set prices.

Economic theory says no such thing. It is educational that some economists should think that there is no mathematical difference between these two versions and mathematicians should think that there is no economic difference and that both should be wrong' (Dorfman et al., 1958, note 1, p. 352).

[4] Therefore, we cannot agree with such a statement as 'Marshall was virtually the first author after Walras clearly and explicitly to derive demand curves from utility functions. In Mathematical Appendix II of his *Principles*, Marshall gives the equilibrium condition for the consumption of commodity x as $MU_x = p_x MU_n$. Taken across all goods this gives the familiar equimarginal rule:

$$MU_x / p_x = MU_y / p_y = MU_z / p_z = MU_n,$$

MU_n being that Marshall calls the marginal utility of money. The 'marginal utility of money' is a confusing phrase because what Marshall had in mind was not the marginal utility of an individual's *stock* of money holdings but the marginal utility of his money income *flow* per limit of time, ssay, a day or a week' [Blaug, 1995, p. 338]. See also Patinkin (1963, p. 104]).