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A Research Framework for Analyzing High Speed Transport Protocols Based on Control-theory

Balázs Sonkoly (Corresponding author), Boglárka Simon, Tuan A. Trinh, Sándor Molnár

High Speed Networks Laboratory

Department of Telecommunications and Media Informatics

Budapest University of Technology and Economics

Magyar tudósok körútja 2, H-1117, Budapest, Hungary

Tel: (361) 463 2664, Fax: (361) 463 3107

E-mail: {sonkoly, simonb, trinh, molnar}@tmit.bme.hu

Abstract

In the recent years, several transport protocols have been proposed for very high bandwidth-delay product networks. However, little is known about the performance of these new mechanisms as well as the interaction with other elements of the network (such as the RED queue management). On the other hand, the control-theoretic approach has proved to be a very useful tool in order to get analytical insight into the performance of congestion control algorithms. In this paper, a control-theoretic research framework is designed and implemented for analyzing high speed transport protocol proposals in network environments with RED active queue management. As a case study, a comprehensive control-theoretic analysis of a promising proposal, namely the HighSpeed TCP is provided. The main contributions of this paper are the following. First, we provide a fluid-flow model for HighSpeed TCP/RED networks. Second, a comprehensive and systematic implementation methodology is described in detail, and a Simulink-based framework is designed for analyzing fluid-based models. Third, we derive a stability condition for HighSpeed TCP/RED networks. The flow-level results are validated by packet-level simulations conducted in Ns-2. Finally, an extension of the framework is presented that makes it capable of describing the interaction of different transport protocols.

Keywords: High speed TCP protocols, RED, control-theory, stability

1. Introduction

Transmission Control Protocol (TCP) has played an important role in the success of Internet. The original protocol providing a reliable, connection-oriented service on top of IP networks dates back to 1981 (RFC 793). In the mid 1980s, serious incidents were experienced in the Internet when the network performance fell down by several orders of magnitudes. This phenomenon, called *congestion collapse*, raised the urgent need of some more sophisticated control mechanism in the transport layer. The original solution for the congestion collapse was provided in [1]. An essential part was added to TCP including the congestion control mechanisms. The congestion management of TCP is composed of two important algorithms. The *Slow-Start* and *Congestion Avoidance* algorithms allow the protocol to increase the data sending rate of sources without overwhelming the network and help to avoid congestion collapse. The protocol updates a variable called *congestion window* (cwnd, *w*) that directly affects the sending rate by means of limiting the number of unacknowledged packets in the network based on a sliding window mechanism which involves a *self-clocking* control. The congestion window variable is adjusted according to various algorithms in different phases of the connection. The basic mechanism was incrementally developed and tuned introducing new additional algorithms, e.g., RTO calculation and delayed ACK in 1989 (RFC 1122), SACK in 1996 (RFC 2018) and NewReno in 2004 (RFC 3782) just to mention a few.

TCP congestion control had managed successfully the stability of the Internet in the past decades but it has reached its limitations in “challenging” network environments. The new challenges of next-generation networks (e.g., high speed communication or the communication over different media) generated an urgent need to further develop the congestion control of the current Internet. In recent years, several new proposals and modifications of the standard congestion control mechanism have been developed by different research groups all over the world. These new mechanisms and TCP versions address different aspects of future networks and applications and improve the performance of regular TCP. For example, standard TCP (*Reno* version) cannot provide acceptable performance in wireless or mobile environments where the propagation delay and the available bandwidth can suddenly change (e.g., during inter-system handover) which can result in multiple back-offs or in extreme cases in disconnection. In order to remedy this problem, new TCP versions have been dedicated to this environment. The drawbacks of standard TCP Reno can be experienced in high speed wide area networks, as well. These networks can be characterized by *high bandwidth-delay product* (BDP) and TCP cannot efficiently utilize them due to its conservative congestion control scheme. As a response to this problem, the research community has proposed several new transport protocols recently referred as *high speed TCPs* or *high speed transport protocols*.

HighSpeed TCP (HSTCP) [2] is specifically designed for use in networks with high bandwidth-delay product, and it is a promising solution due to its incremental improvement that makes the AIMD mechanism an adaptive and more or less scalable algorithm. However, there exist very few studies on the performance implication of HighSpeed TCP so far (see e.g., [3, 4]). Neither its interaction with other elements of the network (such as the RED queue management) is well understood. These issues have motivated our work. Partial results of this work can be found in [5, 6].

In this paper, we provide a control-theoretic model to estimate the performance of HighSpeed TCP in a very high bandwidth-delay product network environment. The motivation behind our approach is to gain analytical insight into the performance of HighSpeed TCP. As it is presented in Section 2.3, control-theoretic approach proves to be a very promising tool to model the dynamics of traditional TCP/AQM networks (TCP/RED network in particular) [7, 8, 9, 10]. The main contributions of this research work are the following. First, we provide a fluid-flow model for HighSpeed TCP/RED networks. Second, a comprehensive and systematic implementation methodology is described in detail and a Simulink-based framework for analyzing fluid-based models is designed and implemented. In this research framework, the HighSpeed TCP/RED model is implemented, however, it can be applied for a wide range of loss-based TCP versions. Third, we give stability conditions for HighSpeed TCP/RED networks that can be used to find adequate parameter settings for RED gateways. The analytical results are validated by packet-level simulations using Ns-2. Finally, an extension of the framework is presented that makes it capable of describing the interaction of different congestion control mechanisms and high speed transport protocols.

The rest of the paper is organized as follows. Section 2 is devoted to present the related work. In Section 3, the fluid-flow model of HighSpeed TCP/RED networks is given. Section 4 and 5 describes the Simulink implementation of the model and validates the results based on packet-level simulations, respectively. In Section 6, a control-theoretic stability analysis for HighSpeed TCP/RED networks is provided. Section 7 is devoted to present an extension of the framework for modeling the interaction of different congestion control schemes. Conclusions are given in Section 8.

2. Related work

This section is devoted to briefly summarize the related work on recent TCP versions and Active Queue Management (AQM) techniques. The most important results on TCP stability analysis are also presented.

2.1 Related work on TCP versions

The huge number of new ideas has resulted in different new TCP versions implemented in several environments. In order to select the “optimal” transport protocol, extensive performance analysis is necessary in a wide range of network environments and applications. In the recent years, many papers were published deepening our understanding of these new protocols regarding performance characteristics, co-existence issues, and other important properties affecting the deployability of them. Here, a brief overview is given on some promising TCP versions. The main properties of the traditional TCP Reno and the TCP variants analyzed in the paper are presented in Table 1 while a more detailed overview can be found in [11].

HighSpeed TCP (HSTCP) [2] is a modification to TCP’s congestion control mechanism for use with TCP connections with large congestion windows. It changes the TCP response function to achieve better performance on high capacity links. HSTCP is based on an AIMD (Additive Increase Multiplicative Decrease) mechanism where the increase and decrease parameters ($a(w)$ and $b(w)$) are functions of the current value of the congestion window (see the corresponding row of Table 1) yielding an adaptive and more or less scalable algorithm. HSTCP introduces a new relation between the average congestion window and the steady-state packet drop (or marking) rate. It is designed to have the standard TCP response in environments with mild to heavy congestion (packet loss rates of at most 10^{-3}) and to have a different, more aggressive response in environments of very low congestion event rate.

Ideas to introduce MIMD (Multiplicative Increase Multiplicative Decrease) mechanisms for TCP have also been considered. Scalable TCP (STCP) [12] is a good example which has been suggested as an efficient transport protocol for high speed networks. Here, the multiplicative increase and multiplicative decrease algorithm guarantees the scalability of the protocol. The congestion window is increased by a constant parameter (a) as a response to a received acknowledgement, while it is reduced in a multiplicative manner (by bw) in case of packet losses (see Table 1). A proposed setting for the constants are $a = 0.01$ and $b = 0.125$ [12].

Table 1. Details of TCP Reno and high speed TCP versions analyzed in the paper

protocol	window adjustment	when	reaction to loss
TCP Reno	$w \leftarrow w + \frac{1}{w}$	per-ACK	$w \leftarrow 0.5w$
HSTCP	$w \leftarrow w + \frac{a(w)}{w}$	per-ACK	$w \leftarrow w - b(w)w$
STCP	$w \leftarrow w + a$	per-ACK	$w \leftarrow w - bw$

In order to solve the TCP severe RTT unfairness problems BIC TCP has been developed [13]. BIC TCP combines two schemes called additive increase and binary search. When the congestion window is large the additive increase with a large increment ensures linear RTT fairness as well as good scalability. Under small congestion windows, binary search increase is designed to provide the required TCP friendliness. Further research with this version has been resulted in CUBIC. CUBIC [14] is an enhanced version of BIC TCP. It simplifies the BIC window control and improves its TCP-friendliness and RTT-fairness. It is worth noting that CUBIC is the default TCP protocol of current Linux kernels. TCP Libra [15] is another solution to provide RTT fairness while maintaining a good friendliness with TCP New Reno. The research to make further improvements yielded to the development of H-TCP [16]. The authors of H-TCP promise that the asymmetry due to the modification in HSTCP and also in STCP can be eliminated with their method. The idea of incorporating accurate bandwidth estimations into the TCP congestion control has also opened a new path in TCP research. TCP-Westwood [17] is a prominent example where eligible rate estimation methods to intelligently set the congestion window and slow-start threshold have been introduced. An interesting solution has been developed in LTCP [18]. LTCP is a two dimensional congestion control. The macroscopic control uses the concept of layering to quickly and efficiently make use of the available bandwidth whereas microscopic control extends the existing AIMD algorithms of TCP to determine the per-ack behavior.

The research on the delay-based ideas has resulted in FAST [19]. FAST has the same equilibrium properties as TCP Vegas [20] but it can also achieve weighted proportional fairness. The delay-based control also appears in other proposals like TCP Africa [21]. TCP-Africa is a hybrid protocol that uses a delay metric to determine whether the bottleneck link is congested or not. In the absence of congestion it uses an aggressive, scalable congestion avoidance rule but in the presence of congestion it switches to the more conservative Reno congestion avoidance rule. Compound TCP [22] is another important example where a synergy of delay-based and loss-based approach has been implemented. It uses a scalable delay-based component into the standard TCP Reno congestion avoidance algorithm, and it is the default transport protocol of Microsoft Windows Vista and Server 2008 products.

2.2 Related work on Active Queue Management (AQM)

In order to get efficient network operation and to avoid congestion collapse, TCP's congestion control mechanism is an important component. However, the dynamic behavior of the network is not solely affected by these mechanisms but network routers also play important roles. A typical network router maintains a set of queues (buffers) belonging to different outputs. Traditional drop-tail routers hold the packets to be scheduled in such queues. When the queue is full, the incoming packets are dropped. By this scheme, the congestion cannot be controlled. Therefore, several Active Queue Management schemes were proposed in order to avoid congestion at the routers by means of efficient buffer management. Packet dropping policies are congestion management mechanisms implemented in the network routers that reactively or proactively drop packets in order to reduce congestion and free up buffer space [23]. An exhaustive review of possible AQM schemes is given in [23] and a detailed classification of them is proposed, as well. This classification is based on the network environment (ATM or IP), the type of congestion management mechanism (avoidance or control and recovery), the number of used thresholds (none or global or per-connection), the state information (global or per-connection), and the queue behavior (static or dynamic). The main goal of packet dropping policies is to avoid or control congestion, however, other performance metrics, such as application throughput, network utilization, fairness performance, and synchronization problems of TCPs, are also significantly affected. Therefore, several research papers deals with the performance analysis of AQM schemes in different network environments (see e.g., [7, 24, 25, 26]).

One of the most important AQM scheme is Random Early Detection (RED) [27] as it is implemented in a wide range of commercial routers, as well. However, other AQM mechanisms were proposed to enhance the capability of the basic RED algorithm, e.g., SRED [28], Adaptive RED [29], REM [30], Blue [31] just to mention a few.

Using RED, in contrast to traditional drop-tail algorithm, packets from the queue can be dropped or marked – if it is used together with the Explicit Congestion Notification (ECN) proposal [32] – according to a marking profile when the buffer is not full. When the average queue size is under a threshold then the incoming packets are accepted. As the average queue size increases, the dropping or marking probability of an arriving packet increases, too (*early* *drop*). Reaching an upper threshold (or the buffer size), the incoming packets are dropped or marked with a probability of 1 (*forced drop*). This mechanism helps to avoid global synchronization which is a known problem of drop-tail queues, and enhances the global power (defined as the ratio of throughput to delay) and fairness characteristics, as well [27].

The basic RED algorithm has three parameters ($p_{max}, t_{min}, t_{max}$) [27] characterizing the packet marking profile. This function and the parameters are shown in Figure 1 and Table 2, respectively. The average queue size which is the input parameter of the marking profile is calculated using a low-pass filter with an exponential weighted moving average (EWMA). The time constant of the filter or the weighting factor (w_q) is also an important parameter of a RED gateway.

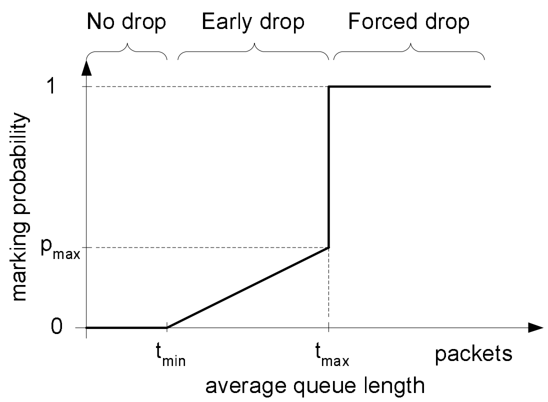


Figure 1. RED packet marking profile

Table 2. RED parameters

p_{max}	maximum probability of early drop
t_{min}	minimum threshold
t_{max}	maximum threshold
w_q or *q_weight*	weighting factor or queue weight

The main drawback of RED is rooted in the careful parameter settings that is required in order to provide good performance. This parameter tuning and the performance evaluation of RED is addressed by several research papers (see e.g., [8, 33, 34, 35]), however, the optimal configuration for different network environments is still an open issue.

2.3 Related work on TCP stability analysis

The current Internet including network routers, links, data sources and sinks, congestion control mechanisms is the largest artificial distributed feedback system. The modeling of such a system is a very challenging mathematical issue. In recent years, several mathematical tools and results have been adopted in order to analyze the dynamic behavior of this network or to provide designing guidelines regarding different network elements. Two types of studies are of fundamental interest [36]. On the one hand, for the analysis of equilibrium properties, optimization-theory and game-theory provide suitable tools. On the other hand, the dynamic characteristics and the stability of the feedback control system can be analyzed based on a control-theoretic approach.

In [37], the first mathematical model and analysis of congestion control algorithms for general topology networks is presented. The resource allocation is considered as a global optimization problem where utility functions are assigned to the sources. A decentralized primal and dual algorithm is given to implement solutions to relaxations of the optimization problems, respectively. In case of the primal algorithm, first-order source dynamics and static link laws are applied. On the other hand, the dual algorithm uses static source control with first-order link dynamics. The model is used to analyze the stability and the fairness of a class of rate control algorithms. A Lyapunov-based proof of global stability is provided in the absence of delays. A similar approach can be found in [38] where a gradient projection algorithm is given for solving the dual problem. In [39], the basic model of [37] is modified and a general rate-based model is given with dynamic sources and dynamic links, as well. The propagation delays (forward and backward direction) are also included in the model. The non-linear model is linearized around an operating point and stability conditions are derived with basic ideas in the proofs. A possible window-based extension of the model is also investigated. In [40], the equilibrium properties of different TCPs and AQM schemes are analyzed based on a duality model. The utility functions are derived for TCP Reno and TCP Vegas, while the RED and REM AQM mechanisms are modeled, as well.

In [41], the behavior of regular TCP (in congestion avoidance) is modeled by jump process driven Stochastic Differential Equations and a fluid model of the traffic is given. A special network centric loss model is used where the loss indications arrive at the sources from the network as a Poisson arrival process. The main deficiency of this model is the independence of the packet loss process and the data flow. In [7], this deficiency is remedied by modeling a complete system, where losses and TCP sending rates are closely coupled. Thus, the model corresponds to a closed loop control system giving rise to a set of coupled differential equations. Simulation results demonstrate that this model is able to capture the dynamics of TCP Reno accurately. In [8], the previous model given by coupled non-linear differential equations is converted to a linear system via the technique of linearization, and the well developed tools in classical linear feedback control theory are applied. With this technique, the stability margins of TCP Reno/RED networks with single bottleneck link can be analyzed and the parameter settings of RED can be examined. A stability condition for this network is given which is validated by packet-level simulations using Ns-2. In [42], improved controllers for AQM routers are provided. Namely, the P and PI controllers are investigated and shown some benefits of PI controller comparing to RED. In [26], a nonlinear stability analysis of a TCP/AQM network regulated by a Proportional controller is carried out and in the case of delay-free marking, the asymptotic stability is proven for all gains. For delayed feedback, a condition of local asymptotic stability is provided with a region of attraction. The basic single link model of [8] is generalized to network case in [9] and [43].

In order to get a deeper understanding of dynamic characteristics of recent congestion control mechanisms and to provide stability conditions for different network scenarios, the control-theoretic approach has proved to be a very useful tool. For example, in [10], the stability analysis of TCP Westwood is carried out, while in [44], a discrete-time model of FAST TCP is introduced and stability properties are derived analytically.

3. Fluid-flow model

This section is devoted to summarize our fluid-flow model that captures the expected (average) transient behavior of a HSTCP/RED network. The model includes a single bottleneck link fed by identical HSTCP sources. Our model is based on the model presented in [7, 8] for TCP Reno/RED networks. We generalize this model for networks carrying HSTCP traffic.

The following variables are used in the rest of the paper:

W	expected TCP window size (packets)
q	expected queue length (packets)
x	expected queue length estimation (packets)
R	round-trip time (RTT) = $q/C+T_p$ (secs)
C	link capacity (packets/sec)
T_p	fix round-trip propagation delay (secs)
N	load factor (number of TCP sessions)
p	probability of packet mark/drop

3.1 HighSpeed TCP source model

The expected (average) dynamics of HSTCP's congestion window in the analyzed network environment, including RED routers, can be described by the following differential equation:

$$\dot{W}(t) = \frac{a(W(t))}{R(t)} - b(W(t)) \cdot W(t) \cdot \frac{W(t-R(t))}{R(t-R(t))} \cdot p(t-R(t)) \tag{1}$$

where the first term corresponds to the additive increase part of the algorithm using the increase parameter $a(W(t))$. This term expresses that the congestion window is increased by $a(W(t))$ packets per one round-trip time. The second term corresponds to the multiplicative decrease part depending on the decrease parameter $b(W(t))$. RED realizes a proportional marking scheme marking packets of flows according to the flows' bandwidth share. Thus, the decrease of the congestion window is weighted by the delayed rate $W(t-R(t))/R(t-R(t))$ and the marking probability $p(t-R(t))$. We emphasize that this differential equation gives an approximation for the expected dynamics of the congestion window.

The main difference between our HSTCP source model and the published TCP Reno model [7] originates from the fact that the increase and decrease parameters of the HSTCP protocol depend on the current value of congestion window $(W(t))$ which yields a more complicated differential equation than the one for TCP Reno.

3.2 Network model

Considering a network with one single bottleneck link fed by identical TCP flows, the expected transient behavior of the queue can be captured by the following differential equation:

$$\dot{q}(t) = N(t)\frac{W(t)}{R(t)} - 1_{q(t)}C \tag{2}$$

where the first term reflects the increase of the queue length according to the arrival rate and the second term reflects the decrease part depending on the service rate. $1_{q(t)} = 1$ if $q(t) > 0$, and zero otherwise. There is an AQM (Active Queue Management) policy associated with this router. We focus on the classical RED (Random Early Detection) [27] policy characterized by a packet discard function $p(x)$ taking the average queue length estimation $x(t)$ as its argument. At packet level simulation, e.g., in Ns-2 [45], the average queue length is computed after every packet arrival applied an exponentially weighted moving average. This can be described by a *difference* equation that can be approximated by the following *differential* equation [7]:

$$\dot{x}(t) = -Kx(t) + Kq(t) \tag{3}$$

where $K = -C\ln(1-\alpha)$ and α is the forgetting factor. This is a first order low pass filter with a cutoff frequency of K. The approximation of the difference equation is based on the assumption that in the steady-state (if the queue is stable) the packet arrival rate is equal to the service rate. Thus, the sampling period can be estimated by the reciprocal of the capacity.

The AQM module acting as a controller (using control-theoretic terms) feeds back the congestion measure (marking or dropping probability) to the TCP senders according to its parameters. The TCP sources as parts of the controlled plant adjust their sending rates based on the experienced marking probability.

The equations (1), (2) and (3) describing the HighSpeed TCP sources and the network dynamics, respectively, form coupled differential equations modeling the HSTCP/RED network. This system of differential equations due to the complex dependence between the variables and containing variable delay in some arguments, is analytically not tractable. Thus, we apply numerical approximation to solve these complex equations.

4. Implementation of the model

As far as we know there is no detailed explanations about the numerical approximation methods used to solve related differential equations modeling TCP/RED networks. In this section, we illustrate our new, *systematic* approach to implement the previously presented system of differential equations.

The system described by non-linear differential equations has been implemented in the Simulink environment of MATLAB. It includes blocks describing the behavior of each part and we get a clear and tractable framework. The basic elements of the model is presented in Figure 2.

The HSTCP source is modeled by the block called “Cwnd dynamics” which captures the behavior of the HSTCP’s congestion window control algorithm. The size of the congestion window W is set according to the packet-marking probability p and the round-trip time R. The marking probability is derived from the instantaneous queue length q by the AQM module, while the current round-trip time is originated from the queue length. The dynamics of the queue length is affected by the current window size, round-trip time and the system load N (number of sources). The input of the system is the load (N_input). The key network variables are saved to MATLAB workspace by the corresponding elements.

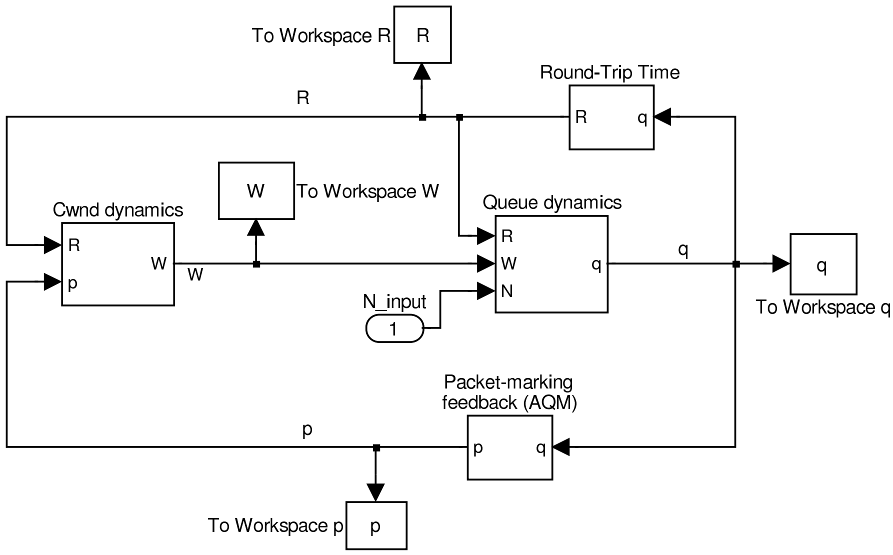


Figure 2. Basic elements of the model (top-level)

4.1 Implementation of HSTCP source model

The building blocks of the subsystem representing the dynamics of HSTCP's congestion window are shown in Figure 3. This module realizes the established differential equation presented in Section 3.1.

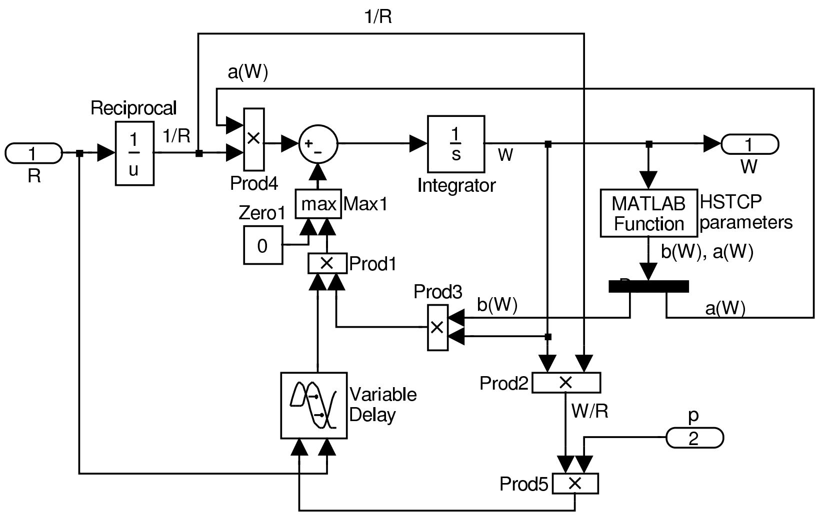


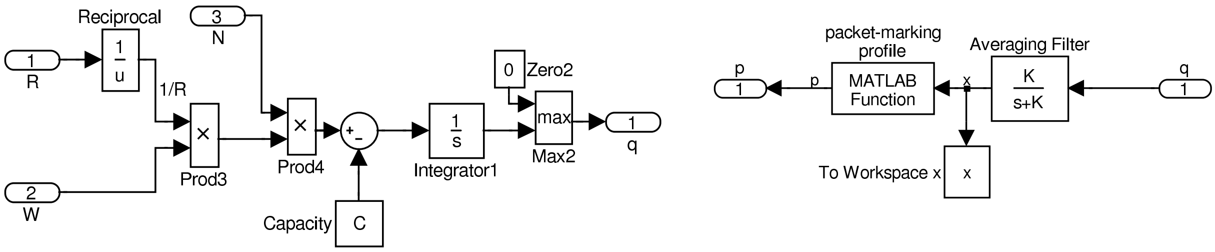
Figure 3. Dynamics of HighSpeed TCP's congestion window

The increase and decrease parameters are derived by an implemented MATLAB function and affects the additive increase and multiplicative decrease algorithms according to the specification of HighSpeed TCP [2].

4.2 Implementation of network model

The block belonging to the single bottleneck queue dynamics is shown in detail in Figure 4(a). It realizes the corresponding differential equation (2).

The module realizing the AQM policy is shown in Figure 4(b). It consists of a low-pass filter averaging the instantaneous queue length and a marking function associated with the RED marking profile (see in Figure 1). The output of this subsystem is the packet marking probability p.



(a) Queue dynamics (b) AQM module

Figure 4. Elements of the network model

The round-trip time module implements a simple correspondence between the queue length and the RTT according to the following equation:

$$R = T_p + \frac{q}{C}. \tag{4}$$

5. Validation of the model

The previously introduced model has been analyzed using Simulink's imbedded differential equation solver tools with different parameters and it has been validated by simulations, as well, conducted in the Ns-2 [45] simulation environment. In this section, the main results of this analysis are presented.

During our investigation, the "ode45" differential equation solver is used which implements the Domand-Prince algorithm (numeric approximation). The investigated network scenario contains identical TCP flows feeding a single queue belonging to the bottleneck link. The parameters of the network scenario used for the validation of the model, namely the network parameters, the parameters of HSTCP and RED (packet-marking profile), are summarized in Table 3. In this work, the default parameter set of HSTCP is used [2], and a single link with high bandwidth-delay product is examined. (We note that in [5], we used another parameter set for the validation.)

Table 3. Parameters of the network scenario used for validation

Network parameters	
capacity	1 Gbps
delay	50 ms
packet size	1 500 bytes

HSTCP parameters	
Low_W	38
High_W	83 000
High_P	10^{-7}
High_Dec	0.1

RED parameters	
p_{max}	0.01
t_{min}	4 000
t_{max}	8 000
$w_q = \alpha$	0.002

As a simple example, the results of a single flow scenario are presented first. We compare the congestion window and the instantaneous queue length of Simulink flow-level simulation (model) with Ns-2 packet-level simulation in Figure 5. Since we focus on the Congestion Avoidance phase, only the steady-state results are shown. (In Ns-2, the Slow-Start mechanism is active from the beginning of a connection until a threshold is reached. This behavior is modeled by an initial condition of the congestion window that is related to that threshold.) It can be observed that the analytic model is capable of accurately capturing the oscillating dynamics of the single HSTCP flow and the bottleneck queue. It is worth noting that an individual TCP-like flow shows oscillation not only at the packet-level but at the flow-level, as well.

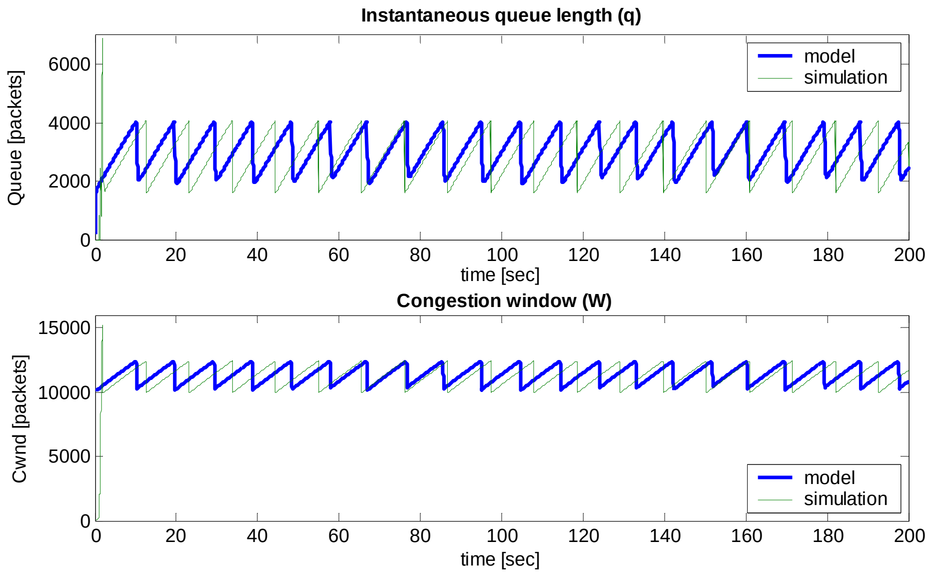


Figure 5. Validation of the model: single HSTCP flow

The model is also analyzed in a more realistic scenario, more specifically, when the bottleneck link is shared among 100 HSTCP flows. Two different settings of queue weight or forgetting factor (w_q or q_{weight} in the packet-level simulation and α in the model) are shown here as an illustration. Figure 6 presents the outcome of the model and the simulation regarding the instantaneous queue length at the bottleneck.

The results are able to validate the accuracy of the model and also emphasize the importance of the appropriate setting of RED parameters. For $\alpha = 0.002$, the bottleneck queue exhibits a (quasi) stable behavior (see the top part of Figure 6), whereas $\alpha = 0.00001$ yields an oscillating system (see the bottom part of Figure 6). In the rest of the paper, we carry out a stability analysis in order to provide an analytical stability condition for HSTCP/RED networks that can be used in RED design.

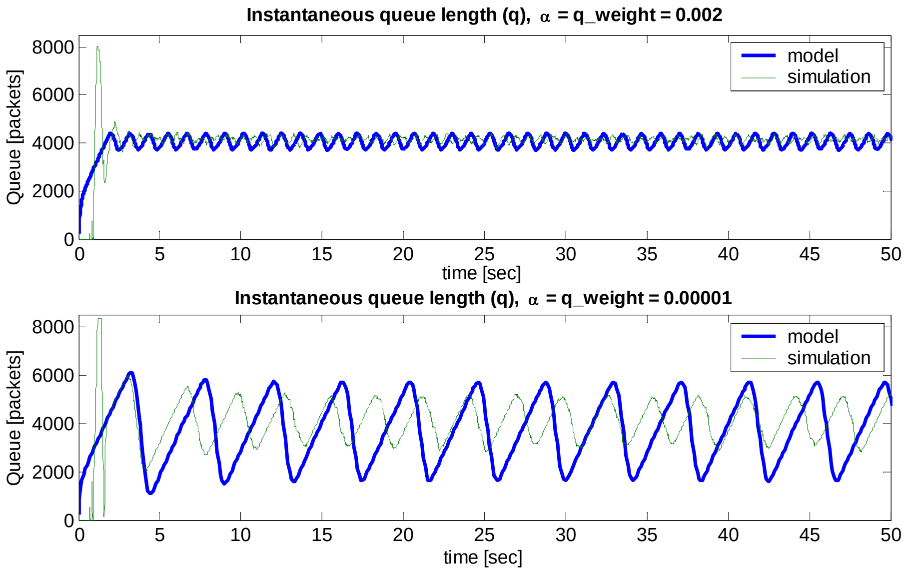


Figure 6. Validation of the model: 100 HSTCP flows, $\alpha = 0.002$ (top) and $\alpha = 1e-5$ (bottom)

6. Stability analysis

In control-system language, the RED module can be referred as the controller or compensator and the rest of the system as the plant. The objective of the controller design is to provide a stable and robust closed-loop system with acceptable transient response. In order to give a stability condition on HSTCP/RED networks, we apply the following methodology. The stability of the system described by our non-linear fluid-flow model (see Section 3) is difficult to analyze directly. Thus, we linearize the model about an equilibrium point first, and the *global asymptotic stability* of the linear system is analyzed. Then we give a condition for RED parameters to stabilize the linear feedback control system for a given range of parameters. Based on the condition, the *local asymptotic stability* of the original system (and the physical system) with reasonable region of attraction can be inferred. Finally, the results are validated and illustrated by packet-level simulations (corresponding to another non-linear system).

A similar stability analysis is carried out for TCP Reno/RED networks in [8].

6.1 Linearization of the model

The plant including HSTCP sources and bottleneck queue is modeled by differential equations (1), (2). Taking W and q as state variables and p as input of the plant, the operating point (W_0, q_0, p_0) is given by $\dot{W} = 0$ and $\dot{q} = 0$. Using small-signal linearization, we assume that load factor and round-trip delay are constants:

$$N(t) \equiv N \quad \text{and} \quad R(t) \equiv R_0 = \frac{q_0}{C} + T_p.$$

Moreover, the increase and decrease parameters of HSTCP can be considered similarly:

$$a(W(t)) \equiv a(W_0) = a_0 \quad \text{and} \quad b(W(t)) \equiv b(W_0) = b_0.$$

a_0 and b_0 can be derived directly from the basic parameters (Low_Window , $High\_Window$, $High\_P$ and $High\_Decrease$) of HSTCP (see [2] for more details):

$$b(W_0) = \frac{(High\_Dec - 0.5) \times (\log W_0 - \log Low\_W)}{\log High\_W - \log Low\_W} + 0.5 \tag{5}$$

$$a(W_0) = \frac{W_0^2 \times p(W_0) \times 2 \times b(W_0)}{2 - b(W_0)} \tag{6}$$

where

$$p(W_0) = \exp\left\{ \frac{\log W_0 - \log Low\_W}{\log High\_W - \log Low\_W} \cdot \left(\log High\_P - \log Low\_P \right) + \log Low\_P \right\}$$

$$Low\_P = \frac{1.5}{Low\_W^2}.$$

Substituting zero for $\dot{W}$ and $\dot{q}$, the following equations can be derived for the operating point:

$$W_0^2 p_0 = \frac{a_0}{b_0} \quad \text{and} \quad W_0 = \frac{R_0 C}{N}. \tag{7}$$

Taking the partial derivatives of the right hand side of equation (1) and (2), we linearize the system about the operating point:

$$\delta\dot{W}(t) = -\frac{a_0 N}{R_0^2 C}\left(\delta W(t) + \delta W(t - R_0) \right) - \frac{b_0 R_0 C^2}{N^2} \delta p(t - R_0) \tag{8}$$

$$\delta\dot{q}(t) = \frac{N}{R_0} \delta W(t) - \frac{1}{R_0} \delta q(t) \tag{9}$$

where the variables denote perturbations ($\delta W \equiv W - W_0$, $\delta q \equiv q - q_0$ and $\delta p \equiv p - p_0$).

This linear system can be transformed into the Laplace transform domain and the transfer function can be derived:

$$P(s) = -e^{-sR_0} P_{hstcp}(s) P_{queue}(s) =$$

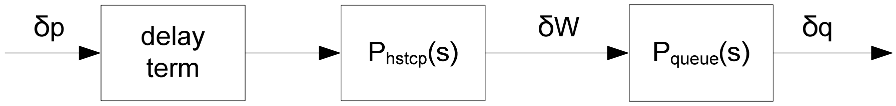
$$= -e^{-sR_0} \frac{\dfrac{b_0 R_0 C^2}{N^2}}{s + \dfrac{a_0 N}{R_0^2 C}\left(1 + e^{-sR_0}\right)} \cdot \frac{\dfrac{N}{R_0}}{s + \dfrac{1}{R_0}} \tag{10}$$

where the first term e^{-sR_0} corresponds to a delay of R_0 and $P_{hstcp}(s)$ and $P_{queue}(s)$ describe the behavior of HSTCP and the queue, respectively. The delay term (e^{-sR_0}) that appears in the denominator of the transfer function $P_{hstcp}(s)$ can be eliminated by a similar approximation as it was used in [8]. The subtle difference to be taken into consideration is the changed condition that makes the approximation acceptable. For regular TCP, this condition requires that $W_0 >> 1$. In our case, the new requirement

$$W_0 >> a_0 \tag{11}$$

is also a reasonable assumption for typical network conditions.

Thus, we get a simplified linear model as it is shown by the block diagram in Figure 7.



$$P_{hstcp}(s) = \frac{\frac{b_0 R_0 C^2}{N^2}}{s + \frac{2a_0 N}{R_0^2 C}} \qquad\qquad P_{queue}(s) = \frac{\frac{N}{R_0}}{s + \frac{1}{R_0}}$$

Figure 7. The simplified linear model

Remarks:

1. The negative eigenvalues of HSTCP and queue dynamics $(-\frac{2a_0 N}{R_0^2 C}, -\frac{1}{R_0})$ indicate that the equilibrium state of the non-linear dynamics is locally asymptotically stable.
2. If we substitute the increase and decrease parameters of HSTCP $a_0 = 1$ and $b_0 = 0.5$ we get the linear model of the network with regular TCP Reno sources.

6.2 *Designing RED for HighSpeed TCP*

The previously presented plant (including the HSTCP sources and the bottleneck queue) and the RED controller form a feedback control system. The transfer function of the RED controller can be modeled in a range of queue length as follows [8]:

$$C(s) = C_{red}(s) = \frac{L}{\frac{s}{K} + 1} \tag{12}$$

where

$$L = \frac{p_{max}}{t_{max} - t_{min}} \quad \text{and} \quad K = -\frac{\ln(1-\alpha)}{\delta} \approx -C\ln(1-\alpha),$$

α is the queue averaging parameter (or forgetting factor) and δ is the sample time which is approximated by $1/C$ in steady-state [7]. L is the slope of the curve characterizing the RED marking profile, whereas K is the cutoff frequency of the RED controller.

The objective of our RED design for a network with HSTCP sources is to select RED parameters L_{hstcp} $(= \frac{p_{max}}{t_{max} - t_{min}})$ and K_{hstcp} to stabilize the feedback control system for a given range of N and R_0. That range can be defined as follows:

$$N \geq N^- \quad \text{and} \quad R_0 \leq R^+.$$

For the sake of clarity, we summarize the used parameters:

L_{hstcp}, K_{hstcp}	:	RED-based control system parameters (explained above)
a_0, b_0	:	HSTCP increase/decrease parameters at the operating point
C	:	Capacity of the link
N^-	:	Minimum number of flows
R^+	:	Maximum value of the RTT

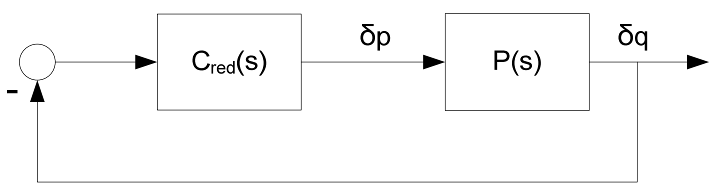
Proposition 1 (Stability condition) *If* L_{hstcp} *and* K_{hstcp} *satisfy*

$$\frac{L_{hstcp} b_0 \left(R^+ C\right)^3}{2a_0 \left(N^-\right)^2} \leq \sqrt{\frac{\omega_g^2}{K_{hstcp}^2} + 1} \tag{13}$$

where

$$\omega_g = 0.1 \min\left\{ \frac{2a_0 N^-}{\left(R^+\right)^2 C}, \frac{1}{R^+} \right\} \tag{14}$$

then, the linear feedback control system showed in Figure 7 is stable for all $N \geq N^-$ *and* $R_0 \leq R^+$.



$$C_{red}(s) = \frac{L_{hstcp}}{\frac{s}{K_{hstcp}} + 1} \qquad P(s) = \frac{\frac{b_0 C^2}{N} e^{-sR_0}}{\left(s + \frac{2a_0 N}{R_0^2 C}\right) \cdot \left(s + \frac{1}{R_0}\right)}$$

Figure 8. Block diagram of the linearized feedback control system

This condition gives a stability region for the HSTCP/RED network. Set of parameters within that region gives stable system.

Proof. The frequency response function of the loop transfer function is

$$L(j\omega)=\frac{L_{hstcp}\dfrac{b_0\left(R_0C\right)^3}{2a_0N^2}e^{-j\omega R_0}}{\left(\dfrac{j\omega}{K_{hstcp}}+1\right)\cdot\left(\dfrac{j\omega}{\dfrac{2a_0N}{R_0^2C}}+1\right)\cdot\left(\dfrac{j\omega}{\dfrac{1}{R_0}}+1\right)}. \tag{15}$$

The goal of this controller design is to force the RED module $(C_{red}(s))$ to dominate closed-loop behavior which is achieved by making the closed-loop time constant $(\approx 1/\omega_g)$ greater than the maximum of the other two time-constants $\left(\dfrac{(R^+)^2C}{2a_0N^-},R^+\right)$ of the transfer function. Thus, the following approximation can be applied

$$L(j\omega)\approx\frac{L_{hstcp}\dfrac{b_0\left(R_0C\right)^3}{2a_0N^2}e^{-j\omega R_0}}{\dfrac{j\omega}{K_{hstcp}}+1}\quad \text{for}\quad \forall\omega\in[0,\omega_g]. \tag{16}$$

For a given range of N and R_0 we get an upper bound for the gain at ω_g:

$$|L(j\omega)|\le\frac{L_{hstcp}\dfrac{b_0\left(R^+C\right)^3}{2a_0\left(N^-\right)^2}}{\sqrt{\dfrac{\omega_g^2}{K_{hstcp}^2}+1}}. \tag{17}$$

From this and (13) it follows that $|L(j\omega_g)|\le 1$ for all $N\ge N^-$, $R_0\le R^+$. Thus, the crossover frequency ω_c (where $L(j\omega_c)=1$) is bounded above by ω_g which yields

$$\angle L(j\omega_c)\ge\angle L(j\omega_g)=\angle\frac{L_{hstcp}\dfrac{b_0\left(R_0C\right)^3}{2a_0N^2}}{\dfrac{j\omega_g}{K_{hstcp}}+1}-\omega_gR_0\ge-90^\circ-0.1\frac{180^\circ}{\pi}>-180^\circ \tag{18}$$

where we used the condition (14). We get that $\angle L(j\omega_c)\ge-180^\circ$ indicating the stability based on Nyquist stability criterion (see e.g., [46]). ■

Remarks:

1. It is important to note that if we substitute the increase and decrease parameters of HSTCP $a_0=1$ and $b_0=0.5$ in (13) and (14) we get the conditions for the network with regular TCP Reno sources [8].

2. In the stability condition, the increase and decrease parameters of HSTCP at the

operating point (a_0 and b_0) can be approximated using the steady-state value of congestion window that can be estimated based on the bandwidth-delay product.

6.3 Numerical examples

In this section, some numerical results are given in order to illustrate the above presented analytical stability condition. Based on the stability condition (Proposition 1) for HSTCP/RED networks, a stability region can be defined. Taking Equations (13) and (14), the RED parameter L_{hstcp} can be expressed in the terms of N^- and R^+ assuming that K_{hstcp} is fixed as follows

$$L_{hstcp} \leq \frac{2a_0\left(N^-\right)^2}{b_0\left(R^+C\right)^3}\sqrt{\frac{\omega_g^2}{K_{hstcp}^2}+1}. \tag{19}$$

The parameters of the network and HSTCP are the same as it is shown in Table 3 while a_0 and b_0 can be approximated based on the bandwidth-delay product and ω_g is given by Equation (14). Therefore, the stability region can be plotted in 3D. In Figure 9, the maximum values of RED parameter L_{hstcp} yielding stable behavior according to the stability condition is plotted in terms of N^- and R^+. Thus, the values of L_{hstcp} below the plotted surface give stable system according to Proposition 1.

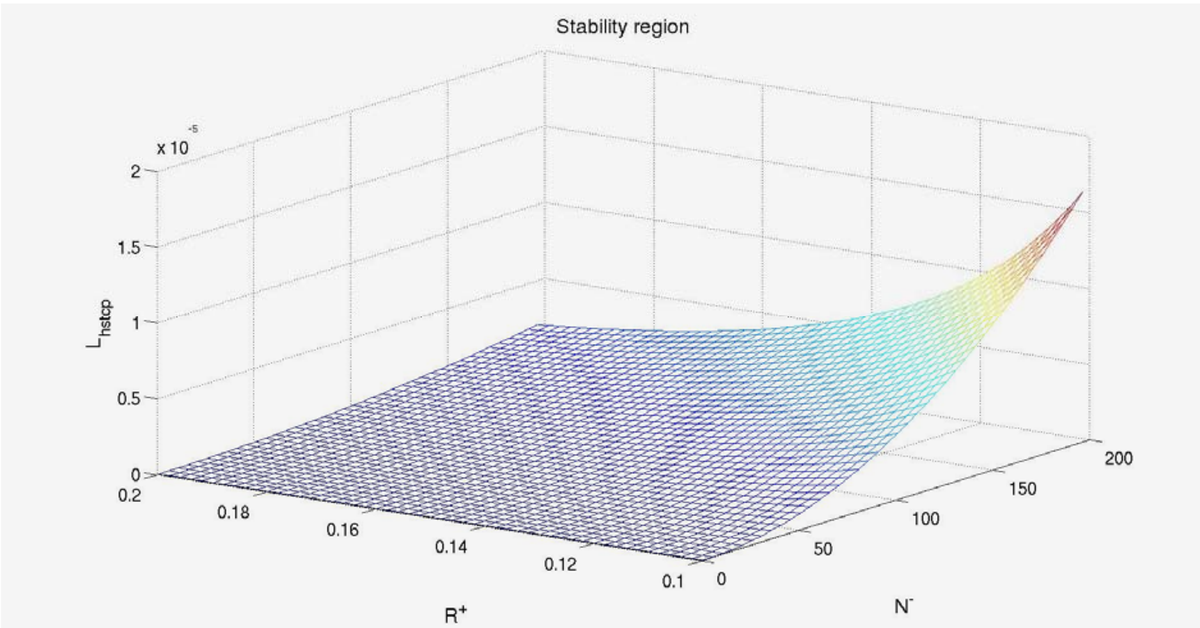


Figure 9. Stability region of HSTCP/RED network

As an illustration, three different scenarios with single bottleneck link are examined and the results of the fluid-flow model and Ns-2 simulations are compared. Here, the RED parameters are fixed ($K_{hstcp} = -C\ln(1-\alpha) = 166.83$, $L_{hstcp} = 2.5\cdot 10^{-6}$), and only the number of flows is varied.

Firstly, a parameter set is chosen that yields an unstable system. The number of HSTCP flows feeding the same queue belonging to the single bottleneck link is $N = 10$. In this case, the stability condition predicts instability. The instantaneous queue length derived from the model and the simulation are shown in the upper part of Figure 10. The steady-state characteristics of the queue dynamics and the *severe oscillation* are well captured by the model, however, the transient behavior shows difference due to the Slow-Start mechanism that is not considered by our model. The permanent oscillation exhibited by the non-linear fluid-flow model regards the expected value of the queue length process. The amplitude of the oscillation does not degrade in time which shows that the asymptotic stability is not satisfied.

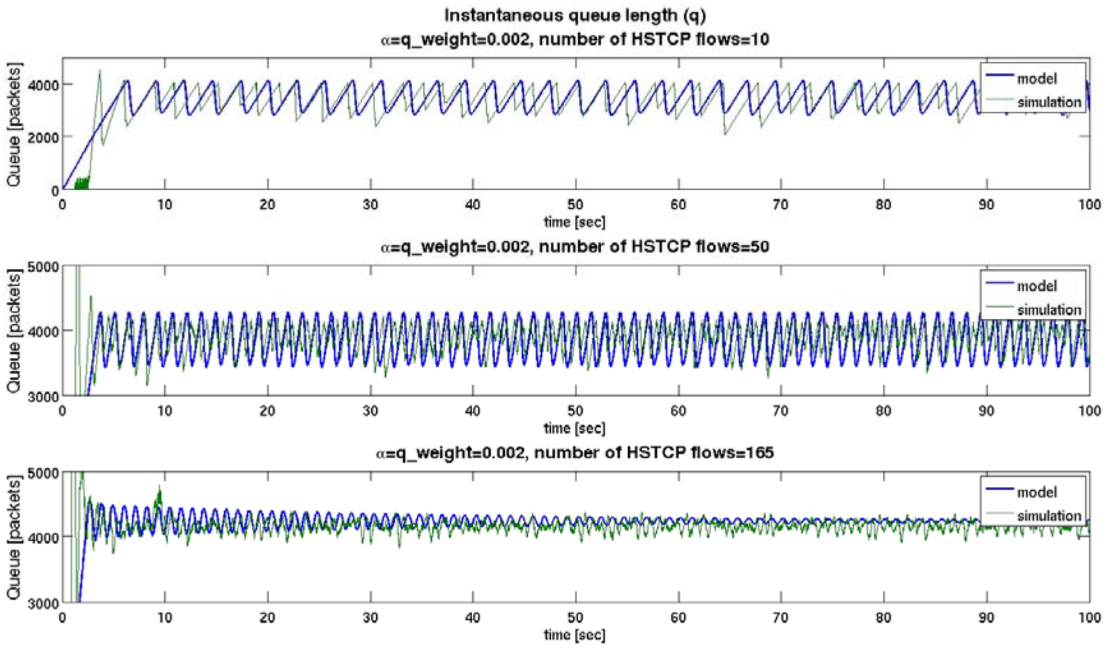


Figure 10. Queue dynamics from model and simulation: 10 flows (top), 50 flows (center), and 165 flows (bottom)

Increasing the flow number, the feedback system moves toward the stable operation. In the second scenario, $N = 50$ is chosen which results in a *moderate oscillation* of queue length as it is shown in the center part of Figure 10. However, the asymptotic stability is not satisfied by the model and the amplitude of the oscillation remains approximately constant in time. More exactly, the oscillation can be observed at a higher frequency and a lower amplitude than previously. We note that the vertical axis of this plot (and the next one) is scaled from 3000 to 5000 in order to provide a better view on the steady-state behavior. According to the stability condition, the limit of the stable behavior (with safe margins) can be observed at around $N = 160$. The bottom part of the figure, shows the queue behavior from the model and the simulation when it is fed by 165 HSTCP flows. The *asymptotic stability* of the steady-state of the model is well demonstrated by the plot as the amplitude of the oscillation is decreasing continually in time until it reaches zero. In the equilibrium state, the expected queue length settles down at an exact queue length. On the other hand, the queue process from the packet-level simulation does not settle down exactly but shows perturbation around the same equilibrium state. The difference in the transient characteristics can be observed in this case, as well.

The stability condition can be used in the design of RED parameters for given network conditions, as well. For example, in the previous scenario, when there exist $N = 50$ HSTCP connections in the network, the closed-loop system shows oscillation. This phenomenon can be compensated by better choice of RED parameters. For this network scenario, the stability condition suggests much lower values of L_{hstcp} than the current one ($L_{hstcp} = 2.5 \cdot 10^{-6}$). Of course, decreasing the gain parameter of RED, we get a slower controller. This gain parameter can be decreased by different ways. One solution is reducing the p_{max} parameter which yields a lighter slope in the marking profile (see Figure 1). Another possibility is expanding the interval of the early drop ($t_{max} - t_{min}$). In the bottom part of Figure 11, $L_{hstcp} = 5 \cdot 10^{-7}$ is achieved by setting p_{max} to 0.002.

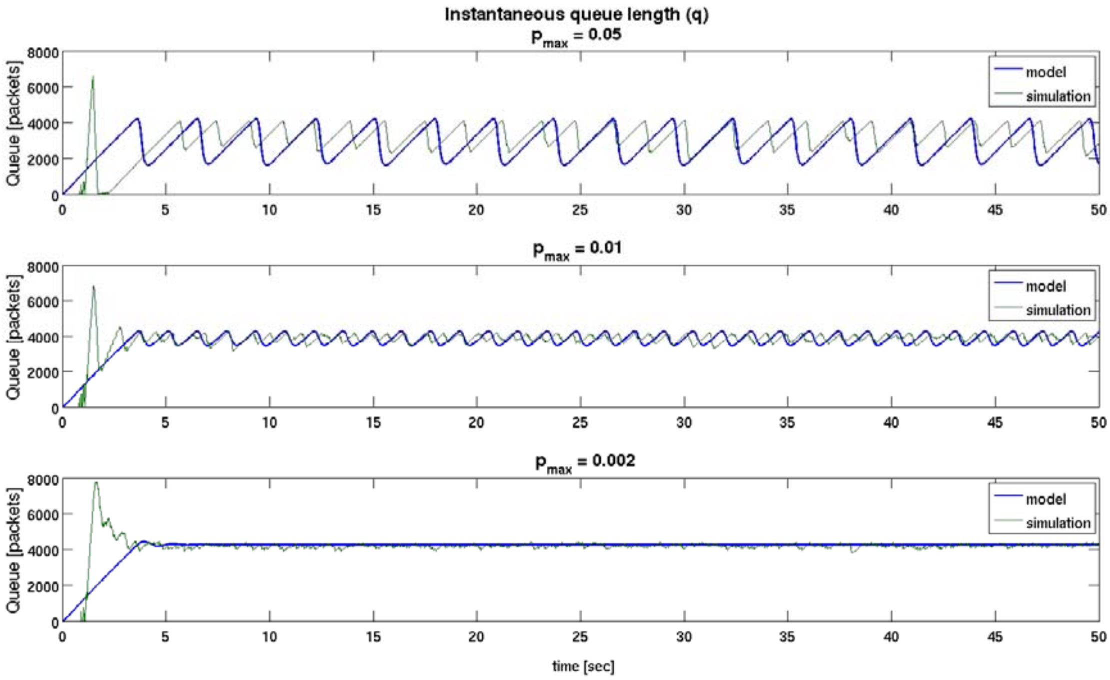


Figure 11. Queue dynamics from model and simulation: $p_{max} = 0.05$ (top), $p_{max} = 0.01$ (center), and $p_{max} = 0.002$ (bottom)

As it is expected from the stability condition, the queue is stabilized by the RED controller. Here, the parameters are chosen to get an asymptotically stable behavior of the model which results in significantly reduced oscillation at the packet-level. In the center part of the figure, the original behavior is presented again while the upper part shows the impact of a non adequate parameter setting ($p_{max} = 0.05$ and $L_{hstcp} = 1.25 \cdot 10^{-5}$) which results in severe oscillation.

7. Extension of the framework

The presented framework can easily be extended with other TCP protocols, as well. For example, the MIMD behavior of an individual Scalable TCP sender can be modeled by a similar way as it has been shown for HSTCP in Section 3.1:

$$\dot{W}(t) = \frac{aW(t)}{R(t)} - bW(t) \cdot \frac{W(t-R(t))}{R(t-R(t))} \cdot p(t-R(t)). \tag{20}$$

The first term of the differential equation describes the multiplicative increase part of the control mechanism while the second term corresponds to the multiplicative decrease mechanism. Here, the increase (a) and decrease (b) parameters are constant values, and the first term (capturing the window increase) explicitly depends on the current value of the congestion window. In [6], we have implemented this model in the Simulink framework and analyzed, as well.

Furthermore, the model is capable of examining the interaction of different congestion control mechanisms at the flow-level. In the top-level model, different TCP algorithms feed the bottleneck queue and the marking probability is fed back to both sources as it is shown in Figure 12. In this model, the number of flows of different mechanisms is the same. In [6], we have presented several results on the intra and inter-protocol fairness properties of HighSpeed TCP (AIMD mechanism) and Scalable TCP (MIMD mechanism). Here, some illustrative results are shown on the inter-protocol fairness characteristics of the AIMD and MIMD mechanisms when the queue at the bottleneck link is regulated by the RED active queue management technique. For a comprehensive fairness analysis of different congestion control schemes operating with traditional drop-tail routers, we refer [11].

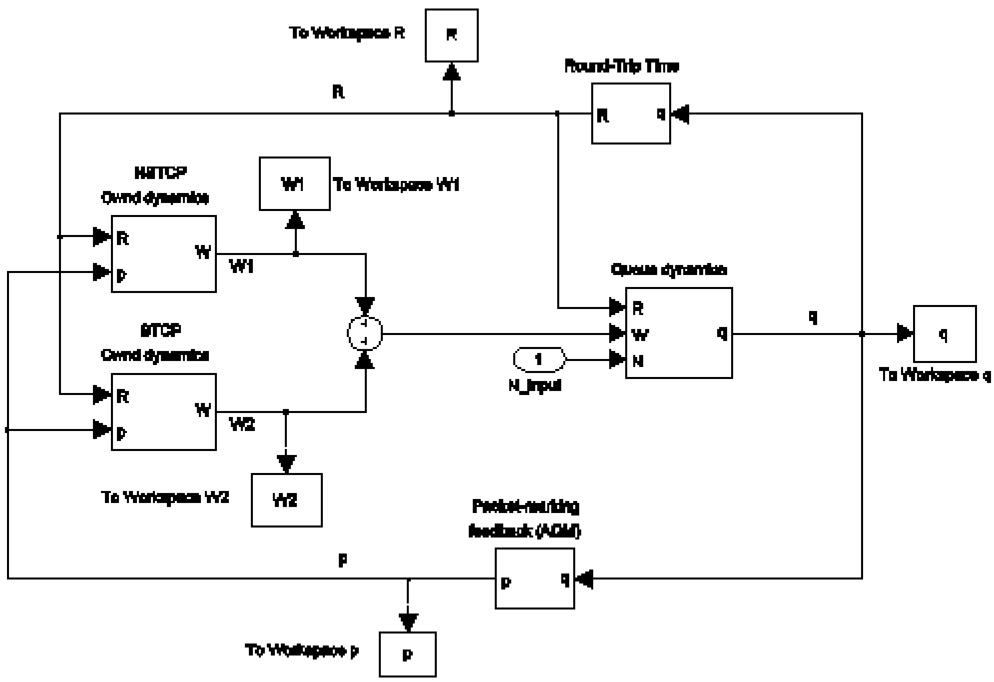


Figure 12. Basic elements of the model (top-level)

The results of the Simulink model (flow-level) and the Ns-2 simulation (packet-level) corresponding to a simple scenario with a single HSTCP and a single STCP flow are presented in Figure 13 and Figure 14, respectively. The figures include the congestion window trajectories and the queue dynamics, as well.

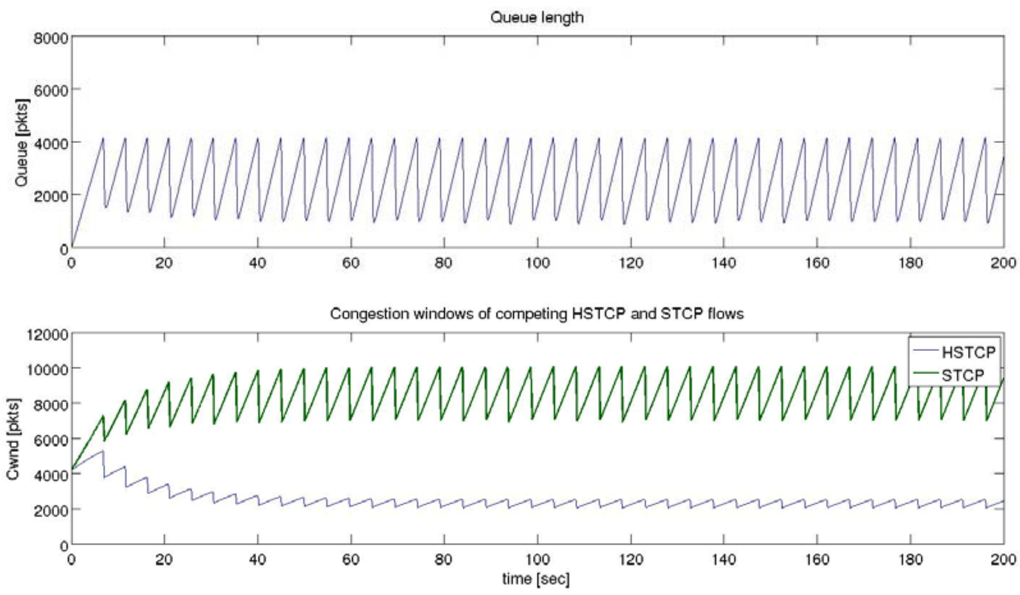


Figure 13: Competing single HSTCP and STCP flows – Simulink model

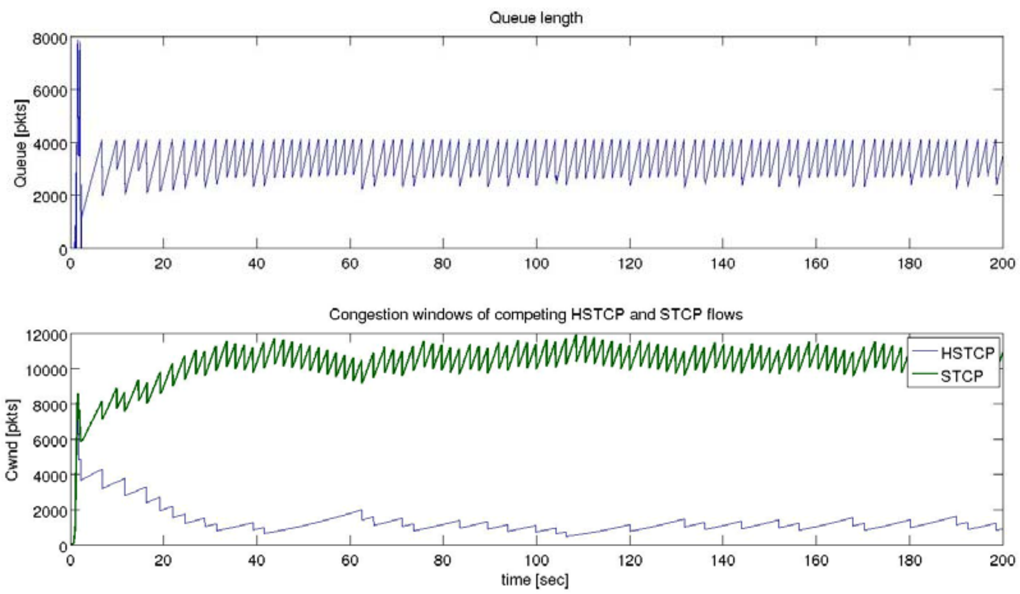


Figure 14: Competing single HSTCP and STCP flows – Ns-2 simulation

It can be observed that after the initial phase (Slow-Start in the packet-level simulation, and initial condition in the model), the interaction of the AIMD and MIMD mechanisms results in an unfair equilibrium state when the HSTCP flow is starved. The model well captures this severe unfairness of the MIMD mechanism operating in a network environment with RED routers.

8. Conclusion

Recently proposed transport protocols are going to play an important role in the congestion control of future networks. However, little is known about the performance of these mechanisms as well as the interaction with other network elements, such as RED active queue management. Applying control-theory in the field of TCP traffic analysis has been proved to be an efficient tool to characterize the flow-level operation and the stability of the protocols.

In this paper, a control-theoretic research framework has been designed and implemented for analyzing transport protocols proposed for very high bandwidth-delay product networks with RED at the routers. As a case-study, a comprehensive control-theoretic analysis of HighSpeed TCP/RED networks has been carried out. We have proposed a fluid-flow model for HighSpeed TCP and investigated its performance in the given network environment. We have described a comprehensive and systematic implementation methodology in detail to get the solutions of the system of non-linear differential equations with given initial conditions. We have also designed and implemented a MATLAB/Simulink-based framework for analyzing fluid-based models applying numerical approximation based differential equation solvers. Based on the model we have derived the stability conditions for HighSpeed TCP network regulated by RED queueing mechanism at the router. The model as well as results have been validated through packet-level simulations by using Ns-2. A possible extension of the framework has also been presented that makes it capable of describing the behavior of competing TCP protocols.

Our analysis raises the issue but it leaves many questions unaddressed. In the future, we plan to extend the current model to study other high speed variants of TCP, such as FAST TCP and XCP. We also plan to use the model to study fairness issues of these protocols in a very high bandwidth-delay product network environment.

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