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Comparison of Decision-Making Methods Comparison of Decision-Making Methods

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Abstract

A comparison is made between the most commonly used decision-making method (AHP), and two methods developed by the authors (APM and CSM). It is shown that the newly developed methods do not change the correlations and types of inequalities between the global priorities of the alternatives when the number of alternatives changes. This differentiates them from the AHP, which does change them. Also, unlike the AHP, which uses pair comparison matrices of criteria relative to goals and alternatives, the developed methods use the creation of pair-comparison matrices of criteria relative to alternatives and of alternatives relative to criteria. In order to obtain a result, it is enough to perform an expert analysis of all the alternatives and one criteria, or otherwise of all the criteria and one alternative. The other pair-comparison matrices are obtained either from the conditions of inverse symmetric matrices, or from the proportionality of weight correlations. The results of the developed methods fully coincide.

Keywords: decision making, hierarchy analysis, matrix, stochastic procedure.

1. Introduction

Presently, problems of multi-criterion choice have a central place in decision-making theory and practice. Taking into account multiple criteria allows us to better approximate situations with our model solutions – a choice of an option/alternative that, in the opinion of experts, most effectively meets all required goals. This makes the study and comparison of decision-making methods very important.

There are three main problems in any decision-making process: the differentiation between options/alternatives, the classification of these options/alternatives and the choice of the best option/alternative.

Many people and organizations are faced every day with problems great and small – everything from the choice of the best candidate for a job position to the choice of how best to perform agrarian reform. These problems are also routinely dealt with in the spheres of politics and the military: places there the number of choices/alternatives is comparatively small, but where the choices/alternatives are themselves incredibly complex. To solve these problems, a wide array of methods is used: the ELECTRE method group; the Podinovsky method; the method of calculation of compromise curves; the Joffrion-Dyer-Feinberg method; the Zeitsman-Vallenius procedure; the Shtoyer method; the STEM method (STEpMethod); methods that use points and curves in visualization; methods of random searching; evolutionary methods; the Analytic Hierarchy Process (AHP) and others.

The most widespread and commonly used method of the choice of an optimal solution based on multiple criteria in the absence of an objective measurement scale is the AHP (Analytic Hierarchy Process). The AHP theory is widely used in many spheres of economics, industry, and in planning everything from individual businesses to entire areas of production.

One of the most serious downsides of the AHP method is that during the change of the number of options/alternatives or criteria, it is possible that there may occur a change in the global priorities of the options/alternatives and criteria. In any case, the ratios between the global priorities will change, which makes it difficult to distribute finances and types of workloads when utilizing several options/alternatives when attempting to reach a given goal.

2. Formulation of the problem

The goal of the present article is the comparison of the AHP with two decision-making methods developed by the authors: the method of the analytical procedure of structurization of a set of alternatives and criteria (APM – analytical procedure method) and the criterion method of analytical stochastic procedures (CSM – criterion stochastic method). These methods, unlike the AHP, do not change the correlation between the global weights of the alternatives and criteria when the number of these alternatives and criteria change.

Let’ start the comparison by applying these methods to a simple example: we have two alternatives A_1 and A_2, and three criteria K_1, K_2 and K_3. We will then continue our analysis by adding a third alternative A_3.

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3. Results of the AHP application

3.1 Example – two alternatives and three criteria

Let's start with the AHP. The pair comparison matrices of the criteria K_1, K_2 and K_3 relative to the goal and the pair-comparison matrices of the alternatives A_1 and A_2 relative to the criteria are presented in the Tables 1-4:

Table 1. Table of pair comparison of criteria in relation to the goal

Goal	K_1	K_2	K_3	V_G
K_1	1	3/4	3/2	1/3
K_2	4/3	1	2	4/9
K_3	2/3	1/2	1	2/9

Table 2. Table of pair comparison of alternatives in relation to the criterion K_1

K_1	A_1	A_2	$V(K_1)$
A_1	1	2	2/3
A_2	1/2	1	1/3

Table 3. Table of pair comparison of alternatives in relation to the criterion K_2

K_2	A_1	A_2	$V(K_2)$
A_1	1	1/2	1/3
A_2	2	1	2/3

Table 4. Table of pair comparison of alternatives in relation to the criterion K_3

K_3	A_1	A_2	$V(K_3)$
A_1	1	3/2	3/5
A_2	2/3	1	2/5

We remind the reader that each line contains the expert comparison of the ratio (weight) of the first structural unit relative to the other ones. This procedure is repeated in each line. Note that the tables correspond to inverse symmetric matrices ($A = \{a_{ij}\},\ i,j = \overline{1,n}$), as in, their elements fulfill the following condition:

$$a_{ij} = \frac{1}{a_{ji}} \tag{1}$$

Here a_{ij} is the matrix element (i – line number, j – column number, both in the matrix and in the corresponding matrix). The eigenvector $V(K)$ of the pair comparison matrix relative to K (the last column of the table) is defined by the following formula:

$$V(K_i) = \frac{\sum_{j=1}^{n} a_{ij}}{\sum_{i,j=1}^{n} a_{ij}} \tag{2}$$

A qualitative measurement scale is in use. This scale is written in the following way: equal importance is 1:1, weak advantage is 3:1, average advantage is 5:1, significant advantage is 7:1, absolute advantage is 9:1 (2,4,6 and 8 are intermediate values of advantages).

The global priorities of the alternative A_1 and A_2 are defined by the products of two matrices: one is a matrix whose columns are the eigenvectors $V(K_j)$, and the other is just the vector-column V_G:

$$W(A) = V(K) * V_G = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{3}{5} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{5} \end{pmatrix} * \begin{pmatrix} \frac{1}{3} \\ \frac{4}{9} \\ \frac{2}{9} \end{pmatrix} = \begin{pmatrix} \frac{68}{135} \\ \frac{67}{135} \end{pmatrix}, W(A_1) = \frac{68}{135} > W(A_2) = \frac{67}{135} \quad (3)$$

From (3), it follows that the alternative A_1 is better-suited to the task at hand than A_2.

3.2 Example – three alternatives and three criteria

Let's add the alternative A_3. In this case, the Tables 2-4 are transformed into the Tables 5-7.

Table 5. Table of pair comparison of alternatives in relation to the criterion K_1

K_1	A_1	A_2	A_3	$V(K_1)$
A_1	1	2	2	1/2
A_2	1/2	1	1	1/4
A_3	1/2	1	1	1/4

Table 6. Table of pair comparison of alternatives in relation to the criterion K_2

K_2	A_1	A_2	A_3	$V(K_2)$
A_1	1	1/2	1	1/4
A_2	2	1	2	1/2
A_3	1	1/2	1	1/4

Table 7. Table of pair comparison of alternatives in relation to the criterion K_3

K_3	A_1	A_2	A_3	$V(K_3)$
A_1	1	3/2	3/5	3/10
A_2	2/3	1	2/5	1/5
A_3	5/3	5/2	1	1/2

Table 8. Pair comparison matrix of criteria relative to the alternative A_1

A_1	K_1	K_2	K_3	$V(A_1)$
K_1	1	3	2	6/11
K_2	1/3	1	2/3	2/11
K_3	1/2	3/2	1	3/11

In this case, the global priorities of the alternatives A_1, A_2 and A_3 are equal to the following:

$$W(A) = V(K) * V_G = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{3}{10} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix} * \begin{pmatrix} \frac{1}{3} \\ \frac{4}{9} \\ \frac{2}{9} \end{pmatrix} = \begin{pmatrix} \frac{31}{90} \\ \frac{7}{20} \\ \frac{11}{36} \end{pmatrix} = \begin{pmatrix} 0.34 \\ 0.35 \\ 0.31 \end{pmatrix},$$

$$W(A_1) = \frac{31}{90} < W(A_2) = \frac{7}{20} \qquad (4)$$

The use of the AHP in the decision-making process has led, after an increase in the number of alternatives, to a change of their initial priorities (A_1 and A_2).

4. Characteristics of the APM and CSM

Let's see how the APM and CSM behave when applied to the decision-making process. Unlike the AHP (pair comparison of criteria relative to a goal), the methods developed by the authors include the construction of pair-comparison matrices of alternatives relative to criteria. In our opinion, these comparisons are more objective than pair-comparisons of criteria relative to a goal. In the developed methods, it is enough to use either all of the criteria and any one alternative, or otherwise to use all alternatives and any criterion. The other tables are reconstructed either from the characteristics of inverse symmetric matrices (as in the APM) or from the proportionality of weight correlations (as in CSM). In order to compare these methods with the AHP, we will use, in these methods, three criteria and one alternative (recall that by "alternative" we mean "option").

5. Results of the APM application

5.1 Example – two alternatives and three criteria

Let's first look at the APM. We will take the same three criteria and the alternative that the experts are in the best agreement on (for instance A_1). To the three Tables 2-4 we will add pair comparisons of criteria relative to the alternative A_1.

We fill the complex table with help from the data present in the Tables 2-4 and 8. Here, x_{ij} are the relative weights of the alternatives and criteria that are determined in the "weighing" process by experts (Table 9).

We then turn the complex table into an ideal inverse symmetric matrix, and then calculate the relative priorities (column V) (Table 10).

Table 9. Complex table ($i = 2, j = 3$)

	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}
x_{11}	1	3	2			
x_{12}	1/3	1	2/3		1/2	
x_{13}	1/2	3/2	1			3/2
x_{21}	1/2			1		
x_{22}		2			1	
x_{23}			2/3			1

Table 10. Final complex table ($i = 2, j = 3$).

	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	V
x_{11}	1	3	2	2	3/2	3	3/10
x_{12}	1/3	1	2/3	2/3	1/2	1	1/10
x_{13}	1/2	3/2	1	1	3/4	3/2	3/20
x_{21}	1/2	3/2	1	1	3/4	3/2	3/20
x_{22}	2/3	2	4/3	4/3	1	2	1/5
x_{23}	1/3	1	2/3	2/3	1/2	1	1/10

The criteria table defines the global priorities of the alternatives and criteria – the sum of the values in the lines and columns respectively. Using V, we fill the lines A_1 and A_2.

Table 11. Criteria table.

	K_1	K_2	K_3	$W(A)$
A_1	3/10	1/10	3/20	11/20
A_2	3/20	1/5	1/10	9/20
$W(K)$	9/20	3/10	1/4	1

The global priorities of the alternatives A_1 and A_2 fulfil the following inequality:

$$W(A_1) = \frac{11}{20} > W(A_2) = \frac{9}{20} \tag{5}$$

5.2 Example – three alternatives and three criteria

We now add the alternative A_3 and fill the complex table with the help of data from the Tables 5-8 (Table 12).

As before, we then turn the complex table into an ideal inverse symmetric matrix, and then calculate the relative priorities (column V) (Table 13).

Table 12. Complex table ($i = 3, j = 3$)

	x_{11}	x_{12}	x_{21}	x_{22}	x_{31}	x_{32}
x_{11}	1		6		3/4	
x_{12}		1		3/4		6
x_{21}	1/6		1	1/8	1/8	
x_{22}		4/3	8	1		8
x_{31}	4/3		8		1	
x_{32}		1/6		1/8		1

Table 13. Final complex table ($i = 3, j = 3$)

	x_{11}	x_{12}	x_{21}	x_{22}	x_{31}	x_{32}	V
x_{11}	1	1	6	3/4	3/4	6	1/5
x_{12}	1	1	6	3/4	3/4	6	1/5
x_{21}	1/6	1/6	1	1/8	1/8	1	1/30
x_{22}	4/3	4/3	8	1	1	8	4/15
x_{31}	4/3	4/3	8	1	1	8	4/15
x_{32}	1/6	1/6	1	1/8	1/8	1	1/30

We then construct the criteria table and calculate the global priorities of the alternatives and criteria:

Table 14. Criteria table

	K_1	K_2	K_3	$W(A)$
A_1	1/5	1/15	1/10	11/30
A_2	1/10	2/15	1/15	3/10
A_3	1/10	1/15	1/6	1/3
$W(K)$	2/5	4/15	1/3	1

$$W(A_1) = \frac{11}{30} > W(A_2) = \frac{3}{10} \tag{6}$$

From (6), it follows that the initial correlations between the priorities of the alternatives A_1 and A_2 (as well as the sign of the inequality between them) did not change, unlike in the AHP.

6. Results of the CSM application

6.1 Example – two alternatives and three criteria

Let's move on to the CSM. From the Tables 2-4, 8, utilizing the proportions of the weight correlations, we reconstruct the pair comparisons of the criteria relative to A_2, and find the vector $V(A_2)$:

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Table 15. Pair comparison matrix of criteria relative to the alternative A_2

A_2	K_1	K_2	K_3	$V(A_2)$
K_1	1	3/4	3/2	1/3
K_2	4/3	1	2	4/9
K_3	2/3	1/2	1	2/9

We then find the transposed matrices $C(A)$ and $C(K)$, which are defined through $V(A)$ and $V(K)$ in the following way:

$$C(A) = V^T(A) * V^T(K) = \begin{pmatrix} \frac{6}{11} & \frac{2}{11} & \frac{3}{11} \\ \frac{1}{3} & \frac{4}{9} & \frac{2}{9} \end{pmatrix} * \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \\ \frac{3}{5} & \frac{2}{5} \end{pmatrix} = \begin{pmatrix} \frac{97}{165} & \frac{68}{165} \\ \frac{68}{135} & \frac{67}{135} \end{pmatrix} \tag{7}$$

$$C(K) = V^T(K) * V^T(A) = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \\ \frac{3}{5} & \frac{2}{5} \end{pmatrix} * \begin{pmatrix} \frac{6}{11} & \frac{2}{11} & \frac{3}{11} \\ \frac{1}{3} & \frac{4}{9} & \frac{2}{9} \end{pmatrix} = \begin{pmatrix} \frac{47}{99} & \frac{80}{297} & \frac{76}{297} \\ \frac{40}{99} & \frac{106}{297} & \frac{71}{297} \\ \frac{76}{165} & \frac{142}{495} & \frac{25}{99} \end{pmatrix} \tag{8}$$

From the system of equations

$$\begin{cases} W(A) = W(A) * C(A) \\ \sum_{i=1}^{2} w(a_i) = 1 \end{cases} \tag{9}$$

We obtain the following values of the global priorities of the alternatives:

$$W(A) = \left(w(a_1), w(a_2)\right) = \left(\frac{11}{20}, \frac{9}{20}\right) \tag{10}$$

Meanwhile, from the system of equations

$$\begin{cases} W(K) = W(K) * C(K) \\ \sum_{i=1}^{3} w(k_i) = 1 \end{cases} \tag{11}$$

We obtain the values of the global priorities of the criteria:

$$W(K) = \left(w(k_1), w(k_2), w(k_3)\right) = \left(\frac{9}{20}, \frac{3}{10}, \frac{1}{4}\right) \tag{12}$$

6.2 Example – three alternatives and three criteria

We add the alternative A_3. From the Tables 5-8, we find the vector $V(A_3)$:

Table 16. Pair comparison matrix of criteria relative to the alternative A_3

A_3	K_1	K_2	K_3	$V(A_3)$
K_1	1	3/2	3/5	3/10
K_2	2/3	1	2/5	1/5
K_3	5/3	5/2	1	1/2

Using the values in the Tables 5-8, 15, 16, we find the matrices $C(A)$ and $C(K)$:

$$C(A) = V^T(A) * V^T(K) = \begin{pmatrix} \frac{6}{11} & \frac{2}{11} & \frac{3}{11} \\ \frac{1}{3} & \frac{4}{9} & \frac{2}{9} \\ \frac{3}{10} & \frac{1}{5} & \frac{1}{2} \end{pmatrix} * \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{3}{10} & \frac{1}{5} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & \frac{31}{110} & \frac{7}{22} \\ \frac{31}{90} & \frac{7}{20} & \frac{11}{36} \\ \frac{7}{20} & \frac{11}{40} & \frac{3}{8} \end{pmatrix} \tag{13}$$

$$C(K) = V^T(K) * V^T(A) = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{3}{10} & \frac{1}{5} & \frac{1}{2} \end{pmatrix} * \begin{pmatrix} \frac{6}{11} & \frac{2}{11} & \frac{3}{11} \\ \frac{1}{3} & \frac{4}{9} & \frac{2}{9} \\ \frac{3}{10} & \frac{1}{5} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{569}{1320} & \frac{499}{1980} & \frac{251}{792} \\ \frac{499}{1320} & \frac{629}{1980} & \frac{241}{792} \\ \frac{251}{660} & \frac{241}{990} & \frac{149}{396} \end{pmatrix} \tag{14}$$

From the system of equations (9) $(i = \overline{1,3})$ we obtain the following values of the global alternatives:

$$W(A) = \big(w(a_1), w(a_2), w(a_3)\big) = \left(\frac{11}{30}, \frac{3}{10}, \frac{1}{3}\right) \tag{15}$$

From the system of equations (11), we obtain the following values of the global priorities of the criteria:

$$W(K) = \big(w(k_1), w(k_2), w(k_3)\big) = \left(\frac{2}{5}, \frac{4}{15}, \frac{1}{3}\right). \tag{16}$$

7. Conclusion

The comparison of the APM and the CSM show a complete correspondence between the results. The change of the number of alternatives does not change the correlation of their initial global priorities, nor does it change the type of inequality between them. The use of the AHP, as we have seen, can lead to a change of the global priorities of the alternatives. The APM and the CSM (in our opinion) use more objective methods, based on the information base of these objects. In the AHP, the pair comparisons of the criteria relative to a goal have a more emotional nature. The APM and the CSM, in the construction of the pair comparison matrices, use similar methods: the characteristics of inverse symmetrical matrices and the proportionality of weight correlations, respectively. In order to determine the global priorities of the alternatives and criteria, it is enough to use all of the pair comparisons by criteria and one alternative or, otherwise, to use all of the pair comparisons by the alternatives and one criterion. Choices are made based on agreement between experts. It should be noted that the proposed CSM increases the number of pair comparison matrices between the alternatives and criteria, but their construction (in our opinion) is much simpler than procedures related to the reconstruction of an ideal symmetrical matrix.

In conclusion, we would like to reiterate that the AHP is one of the most widespread methods of solving a wide range of multicriterion optimization problems. The method and its practical applications can be found in many publications – reviews, monographs and scientific articles. Meanwhile, several scientific journals have discussions about its advantages and disadvantages (“Omega”, “Management Science” and others).

Saati had success in introducing the AHP to people who are not familiar with multicriterion decision –support systems, and who depend solely on expert opinions and their own intuition.

The AHP is presented as a method of quantitative measurement in the comparison scale, and has already been realized in a many computerized decision-support systems, such as Expert Choice.

An unresolved issue in the AHP is the fact that when the number of alternatives or criteria changes, the method can lead to the change of the solution of the problem.

On that note, we would like to remind the reader that the methods proposed in this article do not have this downside: when the number of alternatives or criteria changes, the global priorities of the alternatives or criteria do not change (as well as their correlations). Therefore the solution of the problem does not change either. Problems analysed by this method include performance based budgeting implementation, performance oriented budgeting, performance based program budgeting and others.

Therefore, based on the methods analysed in the article, computerized decision-support systems can be constructed that leave the global priorities of the alternatives unchanged (as well as their correlations). This defines the direction of future studies in this area.

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