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A New Class of Heavy-Tailed Distribution and the Stock Market Returns in Germany

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Abstract

During the past decade, as the fourth largest economy over the world Germany's financial sector plays a key role in the global economy. As one of the most important components of the financial sector, the equity market played a more and more important role. Thus, risk management of its stock market is crucial for welfare of its market participants. To account for the two stylized facts, volatility clustering and conditional heavy tails, we take advantage of the framework in Guo (2017a) and consider empirical performance of the GARCH model with normal reciprocal inverse Gaussian distribution in fitting the German stock return series. Our results indicate the NRIG distribution has superior performance in fitting the stock market returns.

Keywords: normal reciprocal inverse Gaussion, GARCH model, DAX

1. Introduction

As the fourth largest economy over the world, Germany's financial sector plays a key role in the global economy. The country is home to two global systemically important financial institutions, Deutsche Bank AG and Allianz SE, as well as to one of the largest global central counterparties, Eurex Clearing AG. Although the stock market in Germany plays a secondary role compared with the banking system, its impact to the real economy is still significantly material. The Frankfurt Stock Exchange (FWB) is the largest stock change in German and the 10th largest stock exchange in the world by market capitalization. The DAX (Deutscher Aktienindex) is one of the most widely-used stock index and also a blue chip stock market index consisting of the 30 major German companies trading on FWB. The DAX measures the performance of the Prime Standard's 30 largest German companies in terms of the order book volume and market capitalization.

As pointed out by Cont (2001), two of the eleven stylized facts for asset returns are: volatility clustering and conditional heavy tails. The volatility clustering says that different measures of volatility display a positive autocorrelation over several days, which quantifies the fact that high-volatility events tend to cluster in time; and the conditional heavy tails say that even after correcting returns for volatility clustering (e.g. via GARCH-type models), the residual time series still exhibit heavy tails. However, the tails are less heavy than in the unconditional distribution of returns. In this paper, we reconsider the two stylized facts, volatility clustering and conditional heavy tails, but focus on the stock market returns on the Frankfurt Stock Exchange. We take advantage of the model framework in Guo (2017a, 2017b) and are also particularly interested in the normal reciprocal inverse Gaussian (NRIG) distribution, a new class of heavy-tailed distribution, recently emerging in the literature. Guo (2017a, 2017b) compared a variety of heavy-tailed distributions, Student's *t*, Skewed *t*, normal inverse Gaussian (NIG) and NRIG within the generalized autoregressive conditional heteroskedasticity (GARCH) framework for the stock market returns, Guo showed that the NRIG distribution has the best empirical performance in fitting asset returns in the stock markets of the United States and Hong Kong. In this paper, we only focus on the Student's *t* distribution and the NRIG distributions, and our results indicate the NRIG distribution also has superior performance in fitting the stock market returns in German.

1.1 Literature Review

There are many researches in introducing heavy-tailed distributions into the GARCH framework to account for conditional heavy tails (leptokurtosis). For instance, Bollerslev (1987) introduce the Student's *t* distribution and the GARCH model with the Student's *t* distribution could capture dynamics of a variety of foreign exchange rates and stock price indices returns. Politis (2004) introduced the truncated standard normal distribution into the ARCH model and showed the empirical performance of the new type of heavy-tailed distribution on three real datasets. Tavares, Curto and Tavare (2007) model the heavy tails and asymmetric effect on stocks returns volatility into the GARCH framework, and showed the Student's t and the stable Paretian with ($\alpha < 2$) distribution clearly outperform the Gaussian distribution in fitting S&P 500 returns and FTSE returns. Su and Hung (2011) provides a comprehensive analysis of the possible influences of jump dynamics, heavy-tails, and skewness with regard to Value at Risk (VaR) estimates through the assessment of both accuracy and efficiency. Su and Hung consider a range of stock indices across international stock markets during the period of the U.S. Subprime mortgage crisis, and show that the GARCH model with normal, generalized error distribution (GED) and skewed normal distributions provide accurate VaR estimates.

There are also many researches particularly interested in the German stock markets. Lux (2001) provides a statistical analysis of high-frequency recordings of the German share price index DAX. Lux show that one has to go out quite far into the tails for estimation of the extremal index and when considering data at various levels of time-aggregation a test for stability of extreme value behavior over time gives no clear indication of changes of the limiting distribution. Sun, Rachev and Fabozzi (2007) find using high-frequency data that German stocks do exhibit the stylized facts, such as long-range dependence, heavy tails and clustering. Using one of the typical self-similar processes, fractional stable noise, Sun, Rachev and Fabozzi empirically compare this process with several alternative distributional assumptions in either fractal form or *I.I.D.* form (i.e., normal distribution, fractional Gaussian noise, generalized extreme value distribution, generalized Pareto distribution, and stable distribution) for modeling German equity market volatility, and find that fractional stable noise dominates these alternative distributional assumptions both in in-sample modeling and out-of-sample forecasting. Lux (2010) provide statistical analysis for daily returns for 30 German stocks forming the DAX share index as well as the DAX itself during the period

1988–1994. Lux find strong similarity in the extremal behavior of the 30 series and the hypothesis of identical limit laws governing their extreme value distributions is not rejected for all the time series.

In this paper, we follow the model framework in Guo (2017a, 2017b) and are particularly interested in the NRIG distribution, a newly-developed heavy-tailed distribution. Our focuses are on their empirical performance in fitting the stock market returns in German. The remainder of the paper is organized as follows. In Section 2, we discuss GARCH models and the heavy-tailed distributions. Section 3 summarizes the data. The estimation results are in Section 4. Section 5 concludes.

2. The Models

We consider a simple GARCH(1,1) process as:

$$\varepsilon_t = \mu + \sigma_t e_t \tag{2.1}$$

$$\sigma_t^{\ 2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\ 2} + \beta_1 \sigma_{t-1}^{\ 2} \tag{2.2}$$

where the three positive numbers α_0, α_1 and β_1 are the parameters of the process and $\alpha_1 + \beta_1 < 1$. The assumption of a constant mean return μ is purely for simplification and reflects that the focus of the paper is on dynamics of return volatility instead of dynamics of returns. The variable e_t is identically and independently distributed (*i.i.d.*). Two types of heavy-tailed distributions are considered: the Student's *t* and the normal reciprocal inverse Gaussian (NRIG) distributions. The density function of the standard Student's *t* distribution with ν degrees of freedom is given by:

$$f(e_t \mid \psi_{t-1}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}} \left(1 + \frac{e_t^{\ 2}}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \quad \nu > 4. \tag{2.3}$$

where ψ_{t-1} denotes the σ-field generated by all the available information up through time $t-1$.

The NRIG is a special class of the widely-used generalized hyperbolic distribution. The generalized hyperbolic distribution is specified as in Prause (1999):

$$f(e_t \mid \lambda, \mu, \alpha, \beta, \delta) = \frac{(\sqrt{\alpha^2 - \beta^2} / \delta)^{\lambda} K_{\lambda-1/2}(\alpha\sqrt{\delta^2 + (e_t - \mu)^2})}{\sqrt{2\pi}(\sqrt{\delta^2 + (e_t-\mu)^2} / \alpha)^{1/2-\lambda} K_{\lambda}(\delta\sqrt{\alpha^2 - \beta^2})} \exp(\beta(e_t - \mu)), \tag{2.4}$$

where $K_{\lambda}(\cdot)$ is the modified Bessel function of the third kind and index $\lambda \in \mathbb{R}$ and: $\delta > 0$, $0 \le |\beta| < \alpha$. When $\lambda = \frac{1}{2}$, we have the normalized NRIG distribution as:

$$f(\varepsilon_t \mid \psi_{t-1}) = \frac{\alpha K_0(\sqrt{(\alpha^2-1)^2 + \frac{\alpha^2 \varepsilon_t^{\ 2}}{\sigma_t^{\ 2}}})}{\pi \sigma_t} \exp(\alpha^2 - 1). \tag{2.5}$$

3. Data and Summary Statistics

We explore empirical performance of GARCH models with heavy-tailed distribution by using the German stock market returns series. We collected the standardized DAX daily dividend-adjusted close returns from Yahoo Finance for the period from January 4, 1988 to July 19, 2017, covering all the available data in Yahoo Finance. There are in total 7462 observations. Figure 1 illustrates the dynamics of the DAX returns, and the figure exhibits significant volatility clustering.

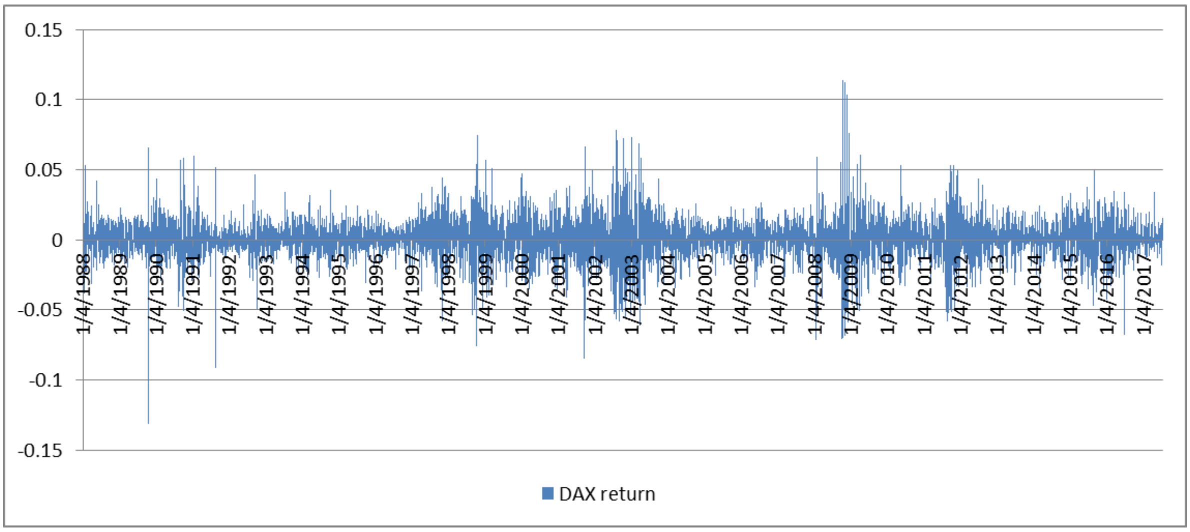

Figure 1. DAX returns

Summary statistics of the data are reported in Table 1. The data present the standard set of well-known stylized facts of asset prices series: non-normality, limited evidence of short-term predictability and strong evidence of predictability in volatility. All series are presented in daily percentage growth rates/returns. The Bera–Jarque test conclusively rejects normality of raw returns in all series, which confirms our assumption that the model selected should account for the heavy-tail phenomenon. The smallest test statistic is much higher than the 5% critical value of 5.99. The market index is negatively skewed and has fat tails. The asymptotic SE of the skewness statistic under the null of normality is $\sqrt{6/T}$, and the SE of the kurtosis statistic is $\sqrt{24/T}$, where T is the number of observations. Almost all series exhibit statistically significant leptokurtosis, suggesting that accounting for heavy-tailedness is more pressing than skewness in modelling asset prices dynamics.

Table 1. Summary statistics. BJ is the Bera-Jarque statistic and is distributed as chi-squared with 2 degrees of freedom, Q(5) is the Ljung-Box Portmanteau statistic, $Q^{ARCH}(5)$ is the Ljung-Box Portmanteau statistic adjusted for ARCH effects following Diebold (1986) and $Q^2(5)$ is the Ljung-Box test for serial correlation in the squared residuals. The three Q statistics are calculated with 5 lags and are distributed as chi-squared with 5 degrees of freedom

Series	Obs.	Mean	Std.	Skewness	Kurtosis	BJ	Q(5)	$Q^{ARCH}(5)$	$Q^2(5)$
DAX	7462	0.04%	1.42%	-0.079*	5.61**	101.2**	9.39*	4.37	37.41**

* and ** denote a skewness, kurtosis, BJ or Q statistically significant at the 5% and 1% level respectively.

We use the Ljung-Box portmanteau, or Q, statistic with five lags to test for serial correlation in the data, and adjust the Q statistic for ARCH models following Diebold (1986). The results that no serial correlation is found for almost all the series confirm our assumption of a constant mean return μ in Equation (2.1). The evidence of linear dependence in the squared demeaned returns, which is an indication of ARCH effects, is significant for all the series.

4. Estimation Results

We study the GARCH(1,1) model with the Student's *t* and the NRIG distributions is defined by maximizing the following log-likelihood function of equation:

$$\hat{\theta} = \arg\max_{\theta} \sum_{t=1}^{T} \log(f(\varepsilon_t \mid \varepsilon_1, \cdots, \varepsilon_{t-1})) \quad . \tag{4.1}$$

Table 2 reports estimation results of the GARCH(1,1) model with the two types of heavy-tailed distribution for all the DAX return series. All the parameters are significantly different from zero. There results show the NRIG distribution has better in-sample performance. Since the two distribution has the same number of parameters, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) also indicate the NRIG distribution has better empirical performance.

Table 2. Estimation of the GARCH model with heavy-tailed innovations

	alpha0	beta1	1/nu (1/alpha)	log-likelihood
Student's t	0.032**	0.917**	0.136**	-12620.1
NRIG	0.041**	0.931**	0.734**	-12549.4

* and ** denote statistical significance at the 5% and 1% level respectively.

5. Conclusion

In this paper, we have examined empirical performance of a newly-developed heavy-tailed distribution, the normal reciprocal inverse Gaussian, under the GARCH framework in fitting the German stock market index returns. We compare it with the most widely-used heavy-tailed distribution, the Student's t distribution. Our results indicate the NRIG has better performance in capture the DAX returns dynamics. Guo (2017a, 2017b) showed the NRIG distribution also performs well in risk management of the US stock return series. We guess the GARCH model with the NRIG distribution would also perform well in risk management of the Germany stock return series. In addition, it would be interesting to consider asymmetric response of conditional volatilities to negative and positive shocks as in Glosten, Jagannathan and Runkle (1993). These are all left for future research.

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